MATH 190, Homework 4

Due date: Wednesday, Nov 14, 2018 (in class)

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your asnwers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwriting please!). Your answers must be ***stapled****, with your name and student number at the top of each page.*

1. (a) Two cyclists are on a north-south straight road. They both start from the same point on the road. Cyclist A rides north at a rate of $2m/sec$ and 7 seconds later cyclist B starts riding south at $1m/sec$. At what rate is the distance separating the two cyclist changing 25 seconds after cyclist A starts riding her bike?

   (b) Now suppose each cyclist is on a different road. Both road are parallel, straight and in the north-south direction and 68 metres apart. Both cyclists start at similar points on each road (that is, the cyclists are 68 metres apart at the start). Cyclist A rides north at a rate of $2m/sec$ and 7 seconds later cyclist B starts riding south at $1m/sec$. At what rate is the distance separating the two cyclist changing 25 seconds after cyclist A starts riding her bike?

2. In a right triangle, the lengths of all sides are changing in such a way that the area of the triangle remains constant and is always equal to $6m^2$. Suppose x and y are the two legs (that is, the two sides that meet at a right angle) and z is the hypotenuse and x is increasing at the rate of $2m/s$. How fast is the hypotenuse changing when $x = 3m$?

3. Consider the function
$$f(x) = -\frac{1}{3}x + 2$$

   (a) Approximate the area under the curve on the interval $[0, 4]$ using Riemann Sums. Use left endpoints and two rectangles ($n = 2$).

   (b) Now approximate the same area using four rectangles ($n = 4$), again with left endpoints.

   (c) Compute the exact area, either by integrating or by drawing a picture and using area formulas. Which approximation is better, (a) or (b)? Are your approximations over or under estimates? Explain why you would expect this at the start of the problem without finding the exact area, perhaps in reference to your picture.

4. Consider the function

$$h(x) = \begin{cases} -2(1-x) - 6 & x \leq 5 \\ -x + 7 & x > 5 \end{cases}$$

   (a) Sketch the graph of $h(x)$.

   (b) Use the graph to compute

$$\int_4^7 h(x)\, dx.$$

   (c) Use integral formulas and rules to compute

$$\int_4^7 h(x)\, dx.$$

5. Compute the following definite integral

$$\int_{-\pi}^{\pi} (\sin x + 2x^4 + 2) dx.$$