HW 5 : Solution (NOT all problems)

(1)

a) $\int \frac{4x+3}{4x^2+6x-1}\,dx = \int \frac{4x+3}{u} \cdot \frac{du}{8x+6}$

Substitution: $4x^2+6x-1 = u$

$(8x+6)\,dx = du$

$\Rightarrow \quad dx = \frac{du}{8x+6}$

$= \int \frac{\cancel{4x+3}}{u} \cdot \frac{du}{2(\cancel{4x+3})}$

$= \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + C$

$\Rightarrow \int \frac{4x+3}{4x^2+6x-1}\,dx = \frac{1}{2}\ln|4x^2+6x-1| + C$

b) $\int 9x^2 \sin(2+6x^3)\,dx = \int \cancel{9x^2}\sin(u)\frac{du}{_2\cancel{18x^2}}$

sub : $2+6x^3 = u$

$18x^2\,dx = du$

$\Rightarrow dx = \frac{du}{18x^2}$

$= \frac{1}{2}\int \sin(u)\,du$

$= -\frac{1}{2}\cos(u) + C$

$= -\frac{1}{2}\cos(2+6x^3) + C$

g) $\int e^x \cos x\,dx = uv - \int v\,du$

Integration by parts:

$\cos x = u \qquad e^x\,dx = dv$

$-\sin x\,dx = du \qquad e^x = v$

$= e^x\cos x - \int e^x(-\sin x)\,dx$

$= e^x\cos x + \int e^x \sin x\,dx$

Another IBP

$\sin x = u \qquad e^x\,dx = dv$

$\cos x\,dx = du \qquad e^x = v$

$\Rightarrow \int e^x \sin x\,dx = e^x\sin x - \int e^x\cos x\,dx$

$$\Rightarrow \int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$\Rightarrow 2\int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\Rightarrow \int e^x \cos x \, dx = \frac{e^x \cos x + e^x \sin x}{2}$$

(2) b) $\displaystyle\int_1^e \frac{\ln x}{x^2}\, dx = \ln x \left(-x^{-1}\right) - \int -x^{-1} \cdot \frac{1}{x}$

IBP : $\ln x = u$ $\quad \frac{1}{x^2} dx = dv$

$\frac{1}{x} dx = du$ $\quad -x^{-1} = v$

$$= -\frac{1}{x} \ln x + \int \frac{1}{x^2}\, dx$$

$$= -\frac{1}{x} \ln x - \int x^{-2}\, dx = -\frac{1}{x} \ln x - \frac{1}{-2+1} x^{-2+1}$$

$$= -\frac{1}{x} \ln x + \frac{1}{x} \Big|_1^e$$

$$= \underbrace{\left(-\frac{1}{e} \cancel{\ln e}^{\,1} + \frac{1}{e}\right)}_{0} - \left(-\frac{1}{1} \cancel{\ln 1}^{\,0} + \frac{1}{1}\right) = -1$$

c) $\displaystyle\int_0^{2\pi} \sin x \ln(\cos x)\, dx = \int_0^{2\pi} \sin x \cdot \ln(u) \cdot \frac{-du}{\sin x}$

sub $\cos x = u$

$-\sin x \, dx = du$

$dx = -\frac{du}{\sin x}$

$$= -\int_0^{2\pi} \ln(u)\, du$$

NOW IBP : Rewrite $\int \ln t \, dt$

$\ln t = u$ $\quad dt = dv$

$\frac{1}{t} dt = du$ $\quad t = v$

$$\int \ln t \, dt = t\ln t - \int \cancel{t} \cdot \frac{1}{\cancel{t}} \, dt = t\ln t - t$$

So far we got:

$$\int_0^{2\pi} \sin x \ln(\cos x)\, dx = -\int_0^{2\pi} \ln(u)\, du$$

$$= -u\ln u + u$$

$u = \cos x$

$$= -\cos x \ln(\cos x) + \cos x \Big|_0^{2\pi}$$

$\ln(1) = 0$

$$= \left(-\cos(2\pi) \cancel{\ln(\underbrace{\cos 2\pi}_{1})}^{0} + \underbrace{\cos 2\pi}_{1}\right)$$

$$-\left(\cos 0 \cancel{\ln(\underbrace{\cos 0}_{1})}^{0} + \underbrace{\cos 0}_{1}\right) = 1 - 1 = 0$$

3. $r(t) = 100\, e^{0.01t}$

Call: Amound of Oil leak out

$$A(t) = \int r(t)\, dt = \int 100 \cdot e^{0.01t}\, dt$$

rate of change in A : $A'(t)$

Sub: $0.01t = u$

$0.01\, dt = du$

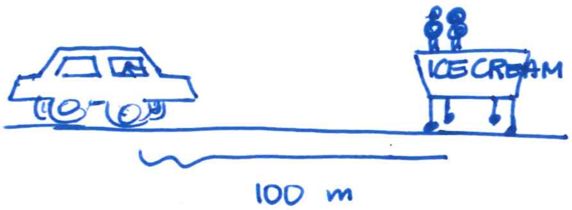
$dt = \dfrac{du}{0.01}$

$$= 100 \int e^u \cdot \frac{du}{0.01}$$

$$= 10000 \int e^u\, du = 10000\, e^{0.01t} + C$$

When $t=0$, NO oil leaked out $\Rightarrow A(0) = 0 \Rightarrow$ use this to find C

$$0 = A(0) = 10000\, \underbrace{\cancel{e^0}}_{1} + C \Rightarrow C = -10000 \Rightarrow A(t) = 10000\left(e^{0.01t} - 1\right)$$



$$a(t) = -50000 \Rightarrow v(t) = \int a(t)\,dt = -50000\,t + C$$

$$\xrightarrow{v(0)=100} \quad 100 = v(0) = -50000 \cdot 0 + C$$

$$\Rightarrow C = 100$$

$$\Rightarrow v(t) = -50000t + 100$$

You stop $\Rightarrow v(t) = 0 \Rightarrow -50000t + 100 = 0$

$$\Rightarrow t = \frac{100}{50000} = \frac{1}{500}\ \text{hr}$$

$$= \frac{36}{5}\ \text{sec} = 7.2\ \text{sec}$$

What is your distance at this time:

$$x(t) = \int v(t)\,dt = \int \left(-50000\,t + 100\right) dt$$

$$= -50000 \cdot \frac{1}{2} t^2 + 100\,t + C$$

$$\xrightarrow{x(0)=0} \quad C = 0$$

$$\Rightarrow x(t) = -25000\,t^2 + 100\,t \quad \text{in Km}$$

$$\xrightarrow[= \frac{1}{500} h]{t = 7.2\text{sec}} \quad x(t) = -25000 \cdot \frac{1}{(500)^2} + 100\left(\frac{1}{500}\right)$$

$$= -0.1 + 0.2 = 0.1\ \text{Km} = 100\ \text{m}$$

on time :)