Justify your answers and show all your work. Unless otherwise indicated simplification of answers is not required.

1. Consider the following function

$$f(x) = -2x + 4\sqrt{x} - \frac{1}{x^2} + \pi.$$

4 marks (a) Compute the derivative of the function $f(x)$.

4 marks (b) Compute the general anti-derivative of the function $f(x)$.

4 marks 2. (a) Compute

$$\int_1^2 \frac{2-x}{\sqrt{x}}\, dx.$$

4 marks (b) Find a function $g(x)$ satisfying $g(\pi) = 1$ such that

$$g'(x) = \sin x + \cos(4x) + e^x.$$

5. Compute the following integrals.

5 marks (a)

$$\int \frac{e^x}{(e^x+1)^2}\,dx$$

5 marks (b)

$$\int_0^{\pi/2} \sin x \cos x\; dx$$

5 marks (c)

$$\int x^2 \ln x \; dx$$

4 marks 7. (a) Approximate the following integral using Riemann Sums

$$\int_1^3 (-2x + 7)\, dx.$$

Use right endpoints and $n = 4$ (ie. four bars).

2 marks (b) Is your approximation less than, greater than, or exactly equal to the true value of the integral? Explain why.

3 marks (c) Sketch the graph of a new function where an approximation with Riemann Sums is exactly equal to the area under the curve.

7 marks 8. The *rate of change* of the height of an elevator is give by

$$r(t) = te^t$$

in meters/second. If at $t = 0$ seconds the elevator is 1 meter off the ground, how high is the elevator after 2 seconds have passed?

4 marks 9. (a) Sketch the graph of a function $f(x)$ satisfying the following two properties:

- $\int_{-2}^{2} f(x)dx = 0$
- $\int_{0}^{4} f(x)dx = 2$

You do not need to find an equation for your function.

4 marks (b) Find two values of b such that

$$\int_{-\pi}^{b} \sin(2x)\ dx = 0.$$

Ensure you justify your answer fully.

5 marks 10. Compute the integral

$$\int_{-1}^{\pi/2} f(x)dx$$

where

$$f(x) = \begin{cases} \cos x & \text{if } x \geq 0 \\ -x^2 + 1 & \text{if } x < 0 \end{cases}.$$