§ 8.3 of the textbook :

(7) $\displaystyle\int_0^1 \frac{x}{e^{3x}}\,dx$

(11) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 7x \cos(3x)\,dx$

(12) $\displaystyle\int 6x \sin(x^2+1)\,dx$

(18) $\displaystyle\int \sqrt{x}\,\ln(x)\,dx$

(16) $\displaystyle\int x^3 \ln(5x)\,dx$

Solution:

In each case, take a moment to think why a certain method is applied and how the calculation will be different if we approach the problem differently.

(7) $\int_0^1 \frac{x}{e^{3x}}\,dx = \int_0^1 x e^{-3x}\,dx \rightsquigarrow$

IBP: $x = u \quad e^{-3x}dx = dv$

$dx = du$

$v = \int e^{-3x}\,dx$

u-substitution $\quad -3x = u$

$-3\,dx = du$

$\Rightarrow dx = \frac{du}{-3}$

$\Rightarrow \int e^u \frac{du}{-3} = -\frac{1}{3}e^u$

$= -\frac{1}{3}e^{-3x}$

$= uv - \int v\,du$

$= -\frac{1}{3}xe^{-3x}\Big|_0^1 - \int_0^1 -\frac{1}{3}e^{-3x}\,dx$

$= -\frac{1x}{3}e^{-3x} + \frac{1}{3}\int_0^1 e^{-3x}\,dx$

$= -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}\Big|_0^1$

$= \left(-\frac{1}{3}\cdot 1\cdot e^{-3} - \frac{1}{9}e^{-3}\right) - \left(\cancel{-\frac{1}{3}0e^0}^{\,0} - \frac{1}{9}\underbrace{e^{0}}_{1}\right)$

$= -\frac{1}{3}e^{-3} - \frac{1}{9}e^{-3} + \frac{1}{9}$

(11) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 7x\cos(3x)\,dx = 7x\cdot\frac{1}{3}\sin(3x) - \int \frac{1}{3}\sin(3x)\cdot 7\,dx$

IBP:

$7x = u \qquad \cos(3x)\,dx = dv$

$7\,dx = du \qquad \int\cos(3x)dx = v$

substitution $\quad 3x = u$

$3dx = du$

$dx = \frac{du}{3}$

$\Rightarrow \int \cos(u)\frac{du}{3} = \frac{1}{3}\sin(3x)$

$= \frac{7}{3}x\sin(3x) - \frac{7}{3}\int \sin(3x)\,dx$

substitution: $3x = u$

$dx = \frac{du}{3}$

$\int \sin(u)\frac{du}{3} = \frac{-1}{3}\cos(3x)$

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$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 7x\cos(3x)\,dx = \frac{7}{3}x\sin(3x) + \frac{7}{9}\cos(3x)\Bigg|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \frac{7}{3}\cdot\frac{\pi}{2}\overset{-1}{\cancel{\sin\left(\frac{3\pi}{2}\right)}} + \frac{7}{9}\overset{0}{\cancel{\cos\left(\frac{3\pi}{2}\right)}} - \frac{7}{3}\cdot\frac{\pi}{3}\overset{0}{\cancel{\sin\left(3\frac{\pi}{3}\right)}} + \frac{7}{9}\overset{-1}{\cancel{\cos\left(3\cdot\frac{\pi}{3}\right)}}$$

$$= -\frac{7\pi}{6} - \frac{7}{9}$$

(12) $\int 6x\sin(x^2+1)\,dx = \int \overset{3}{\cancel{6x}}\sin u \cdot \frac{du}{\cancel{2x}}$

Substitution: $x^2+1 = u$

$2x\,dx = du$

$dx = \frac{du}{2x}$

$$= 3\int \sin u\,du$$

$$= -3\cos u + C = -3\cos(x^2+1) + C$$

(18) $\int \sqrt{x}\ln x\,dx = uv - \int v\,du$

IBP:

$\ln x = u$ $\qquad \sqrt{x}\,dx = dv$ $\quad (\sqrt{x} \to x^{\frac{1}{2}})$

$\frac{1}{x}dx = du$ $\qquad \frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} = v$

$\qquad \frac{2}{3}x^{\frac{3}{2}} = v$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \int \frac{2}{3}x^{\frac{3}{2}}\cdot\frac{1}{x}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\cdot\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + C$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$$

(16) $\int x^3\ln(5x)\,dx = \frac{1}{4}x^4\ln(5x) - \int \frac{1}{4}x^4\cdot\frac{1}{x}\,dx$

$\ln(5x) = u$ $\qquad x^3\,dx = dv$

$\frac{1}{5x}\cdot 5\,dx = du$ $\qquad \frac{1}{4}x^4 = v$

$\frac{1}{x}dx = du$

$$= \frac{1}{4}x^4\ln(5x) - \frac{1}{4}\int x^3\,dx$$

$$= \frac{1}{4}x^4\ln(5x) - \frac{1}{16}x^4 + C$$