Exam Practice : (Riemann Sum & Integral of piecewise functions)

(1) a) Sketch the function

$$g(x) = \begin{cases} e^{x} & x \leq 0 \\ \frac{1}{2}x + 1 & x > 0 \end{cases}$$

(b) Evaluate $\displaystyle\int_{-1}^{2} g(x)\,dx$

Clicker:

A. $1 - e$

B. $4 - \frac{1}{e}$

C. $3 + \frac{1}{e}$

D. $4 + \frac{1}{e}$

(c) Without computing the Right Riemann Sum on the interval $[-1, 2]$, determine if it will be an underestimate or overestimate of the exact value.

Clicker:

A. Over estimate

B. Underestimate

C. Can NOT be determined.

(d) Compute the right Riemann sum for the function $g(x)$ on $[-1, 2]$ with 3 rectangles.

Clicker:

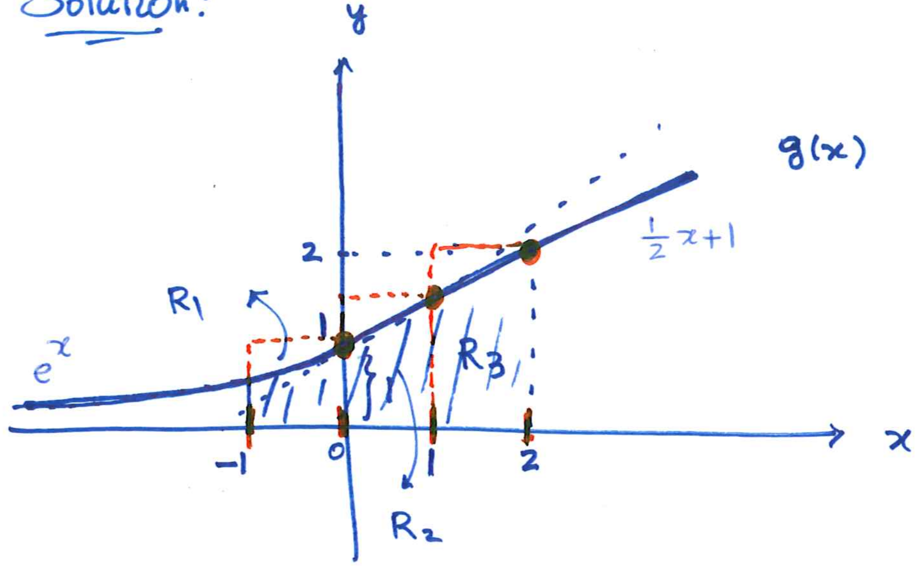
A. $3/2$

B. $3$

C. $\frac{9}{2}$

D. $\frac{7}{2}$

Solution:




$$y = \frac{1}{2}x + 1$$

$$x = 0 \Rightarrow y = 1$$

$$x = 2 \Rightarrow y = \frac{1}{2} \cdot 2 + 1$$

$$= 1 + 1 = 2$$

(b) $$\int_{-1}^{2} g(x)\,dx = \int_{-1}^{0} e^x\,dx + \int_{0}^{2} \left(\frac{1}{2}x + 1\right) dx$$

$$= e^x \Big|_{-1}^{0} + \frac{1}{2} \cdot \frac{1}{1+1} x^{1+1} + x \Big|_{0}^{2}$$

$$\frac{1}{4}x^2 \qquad + x$$

$$= \left(e^0 - e^{-1}\right) + \left(\frac{1}{4} \cdot 2^2 + 2 - 0\right)$$

$$= 1 - \frac{1}{e} + 1 + 2$$

$$= 4 - \frac{1}{e}$$

(c) The area of the rectangles is larger than the actual area

$\Rightarrow$ Over estimate

(d) $[-1,2]$

$n = 3$ $\Rightarrow \Delta x = \dfrac{b-a}{n} = \dfrac{2-(-1)}{3} = \dfrac{3}{3} = 1$

$R_1 = 1 \times 1 = 1$

$R_2 = 1 \times \left(\frac{1}{2} \cdot 1 + 1\right) = \frac{3}{2}$

$R_3 = 1 \times \left(\frac{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} + 1\right) = 2$

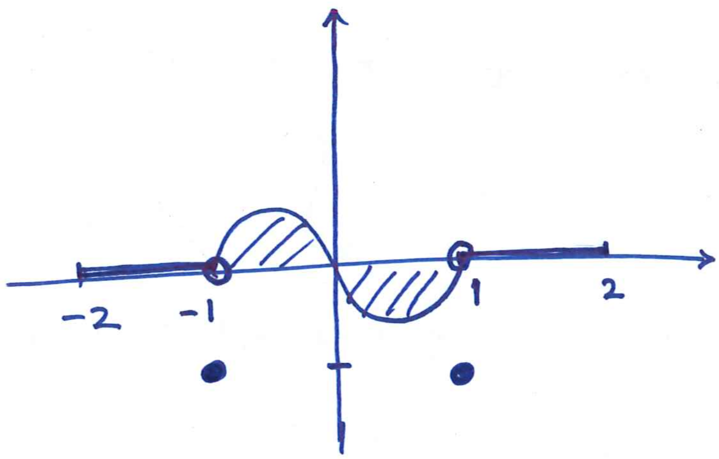
$\Rightarrow$ Area $\approx 1 + \frac{3}{2} + 2 = 3 + \frac{3}{2} = \frac{9}{2}$

(2) Draw a function satisfying the following properties:

(a) $\int_{-1}^{1} f(x)\,dx = 0$

(b) $\int_{-2}^{-1} f(x)\,dx = \int_{1}^{2} f(x)\,dx$

(c) $f(-1) < 0$

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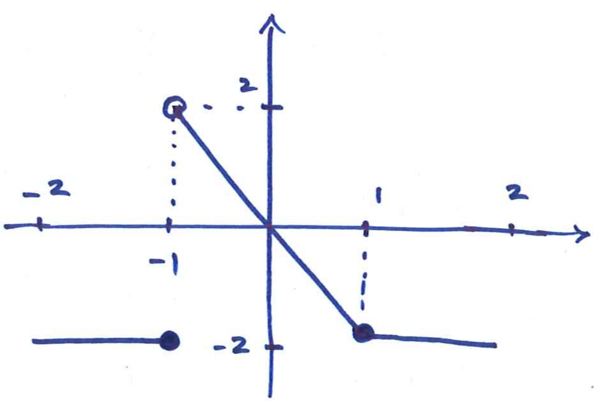


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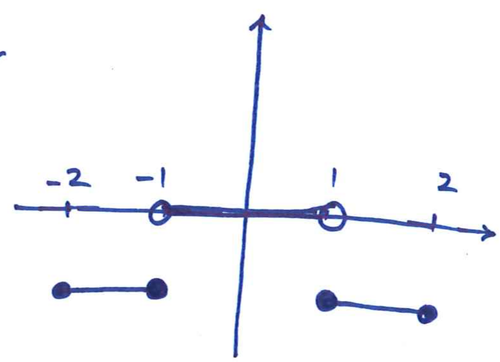


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