Announcements :

- Final Exam : December 14 at 12:00 pm

  Location : MATH 100

  Duration : 2.5 hours

- Office Hours during the exam period:

  Thursday, Dec 6, 11 am – 12 pm in MATx 1118

  in LSK 300 { Thursday, Dec 13, 11 am – 1 pm & 2–3 pm
  Friday, Dec 14, 10 – 11 am

  Change of location in my ealier email.

- Exam topics : Everything week 1 – week 13

  Heavier on Integral Calculus.

- Check everything posted and solve all the examples & problems in :

  → Lecture Notes

  → Labs

  → Quizzes

  → HW

  → Sample exam & review practice

  → Textbook problems

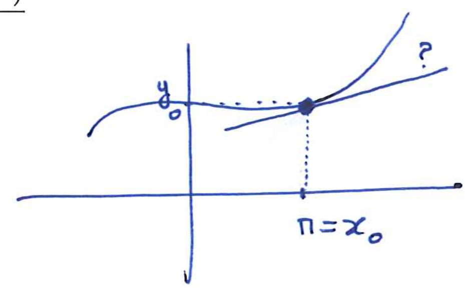
  You should be able to do them without checking the solution.

- $\boxed{\text{TEACHING EVALUATION}}$ → please complete the survey :)

5 marks 3. Find the equation of the tangent line to

$$f(x) = \frac{\cos(2x)}{x}$$

at the point $x = \pi$.



General Equation of a line:

$$y - y_0 = m(x - x_0)$$

? ? ? π

tangent line

$\hookrightarrow m = f'(\pi)$

$y_0 = f(\pi)$

$$f(x) = \frac{\cos(2x)}{x} \Rightarrow f'(x) = \frac{(\cos(2x))' \cdot x - (\cos(2x)) \cdot \overset{1}{\cancel{(x)'}}}{x^2}$$

- $(\underbrace{\cos}_{\text{out}}(\underbrace{2x}_{\text{in}}))' = 2 \cdot -\sin(2x)$

$$\Rightarrow f'(x) = \frac{-2\sin(2x) \cdot x - \cos(2x)}{x^2}$$

$$\Rightarrow f'(\pi) = \frac{-\overset{0}{\cancel{2\sin(2\pi)\cdot\pi}} - \overset{1}{\cancel{\cos}}(2\pi)}{\pi^2} = \frac{-1}{\pi^2} = m$$

- $y_0 = \frac{\cos(2\pi)}{\pi} = \frac{1}{\pi}$

$x_0 = \pi$

$$\Rightarrow \boxed{y - \frac{1}{\pi} = -\frac{1}{\pi^2}(x - \pi)}$$

$$\Rightarrow y = -\frac{1}{\pi^2}x + \frac{1}{\pi} + \frac{1}{\pi}$$

$$\Rightarrow \boxed{y = -\frac{1}{\pi^2}x + \frac{2}{\pi}}$$

5 marks 4. Find the derivative of

$$f(x) = xe^{2x}\sin(x^2).$$

$$(fgh)' = f'gh + g'fh + h'fg$$

$$(x)' = 1$$

$$(\underbrace{e}_{\text{outside}}{}^{\underbrace{2x}_{\text{in}}})' = 2 \cdot e^{2x}$$

$$(\underbrace{\sin}_{\text{out}}\underbrace{(x^2)}_{\text{in}})' = 2x \cdot \cos(x^2)$$

$$f'(x) = 1 \cdot e^{2x}\sin(x^2) + 2e^{2x} \cdot x\sin(x^2)$$

$$+ 2x\cos(x^2) \cdot xe^{2x}$$

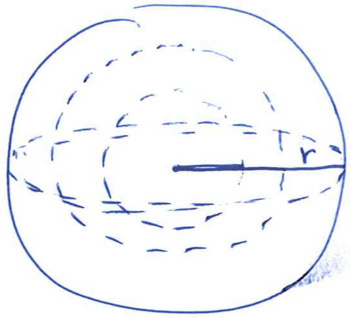
5 marks 6. A spherical snow ball is melting such that its surface area is decreasing at a rate of $0.5\text{cm}^2/\text{min}$. How fast is the volume decreasing when the radius is 6cm? The Volume and Surface Area of a sphere are given by

$$V = \frac{4}{3}\pi r^3 \quad \text{and} \quad A = 4\pi r^2$$

respectively.

Steps:

1) Diagram and label the picture.
2) Read the question carefully & summarize the info.
   - changing quantities
   - Constant "
   - given & unknown rate of change.
3) Relate the variables : 1) Pythagorean
   2) Sin / Cos / tan
   3) Use the given equations in the problem.
4) Differentiate
5) Substitute the info.



Changing quantities:

A = Surface area

V = volume

r = radius

given / unknown info:

$$\begin{cases} \dfrac{dA}{dt} = -0.5 \ \ cm^2/min \\[2ex] \dfrac{dV}{dt} = ? \rightsquigarrow \text{must be negative} \end{cases}$$

when $r = 6$ cm

$V_{(t)} = \frac{4}{3}\pi r_{(t)}^3$, $A_{(t)} = 4\pi r_{(t)}^2$

derive

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3\overset{6}{r}{}^2 \cdot \frac{dr}{dt} \quad \text{use A to find } \frac{dr}{dt}$$

? ?

$$\frac{dA}{dt} = 4\pi \cdot 2r \cdot \frac{dr}{dt} \ ?$$

$$-0.5 = 4\pi \cdot 2 \cdot 6 \cdot \frac{dr}{dt}$$

$$\frac{-0.5}{48\pi} = \frac{dr}{dt}$$

$$\frac{dV}{dt} = \frac{\overset{1}{\cancel{4}}}{\cancel{3}}\cancel{\pi} \cdot \cancel{3} \cdot (\overset{3}{\cancel{6}})^{\cancel{2}} \cdot \frac{-0.5}{\cancel{48}\cancel{\pi}} \quad \begin{matrix} \cancel{8} \\ \cancel{2} \\ 1 \end{matrix}$$

$$= 3 \cdot (-0.5)$$

$$= -1.5 \ cm^3/min$$