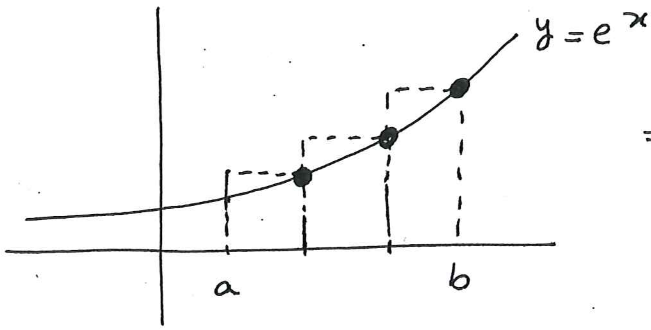
Quiz 5 Solution :

Q1:

(a) False. Check the graph



$\Rightarrow$ This is an over-estimate.

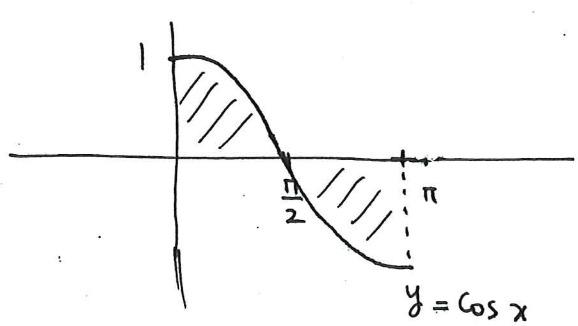
(b) True.

$$\int_0^{\pi} (\cos x + 2)\, dx = \sin x + 2x \Big|_0^{\pi}$$

$$= (\sin \pi + 2\pi) - (\sin 0 + 2\cdot 0)$$

$$= 2\pi$$

Or using the graph of $y = \cos x$, we see that the area under the curve of $\cos x$ from 0 to π is 0.



So only $\int_0^{\pi} 2\, dx = 2x \Big|_0^{\pi} = 2\pi$

Q2:

(a) $f(x) = 3x^{-\frac{1}{2}} - 4e^{x} + 5x^{2} - 1$

anti derive

$$F(x) = 3 \cdot \frac{1}{-\frac{1}{2}+1} x^{-\frac{1}{2}+1} - 4e^{x} + 5 \cdot \frac{1}{2+1} x^{2+1} - x + C$$

$$= 3 \cdot \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} - 4e^{x} + 5 \cdot \frac{1}{3} x^{3} - x + C$$

$$= 6\sqrt{x} - 4e^{x} + \frac{5}{3} x^{3} - x + C$$

(b) We want $F(x)$ such that $F(0) = 1$

From above:

$$F(0) = \cancel{6\sqrt{0}} - 4\cancel{e^{0}}^{1} + \cancel{\frac{5}{3} 0^{3}} - \cancel{0} + C = 1$$

$$\Rightarrow \quad -4 + C = 1 \Rightarrow C = 5$$

$\Rightarrow$ A particular F: $\boxed{F(x) = 6\sqrt{x} - 4e^{x} + \frac{5}{3}x^{3} - x + 5}$