

MATH 190, QUIZ 2

Oct 10, 2018

Time: 15 minutes

Show all your work. No calculators, no books/notes are allowed.

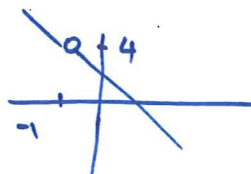
Name (please print): _____

Student number: _____

1. [5 points] Determine whether the following statements are True or False. Provide a justification or a counter-example (an example that contradicts the statement).

(a) $\lim_{x \rightarrow -1} f(x) = 4$ implies that $f(-1) = 4$. *False. limit is only about the value of the function when x is close to -1 NOT exactly at -1 .*

Counter-example



$f(-1)$ is undefined but $\lim_{x \rightarrow -1} f(x) = 4$

(b) If $g(x) = \begin{cases} \frac{x^2 - 6x + 8}{x - 4} & x < 4 \\ \sqrt{x} & x \geq 4 \end{cases}$, then $\lim_{x \rightarrow 4} g(x)$ does NOT exist.

$$\lim_{x \rightarrow 4^+} g(x) = \lim_{x \rightarrow 4^+} \sqrt{x} = \sqrt{4} = 2$$

$\lim_{x \rightarrow 4} g(x) = 2$ False

$$\lim_{x \rightarrow 4^-} g(x) = \lim_{x \rightarrow 4^-} \frac{x^2 - 6x + 8}{x - 4} = \frac{0}{0} \rightsquigarrow = \lim_{x \rightarrow 4^-} \frac{(x-4)(x-2)}{(x-4)} = \lim_{x \rightarrow 4^-} x - 2 = 2$$

2. [4 points] Find the vertical and horizontal asymptotes of $f(x) = \frac{x^2}{x - \pi}$.

$x = \pi$: candidate for V.A.

$$\lim_{x \rightarrow \pi^+} \frac{x^2}{x - \pi} = \frac{\pi^2}{0^+} = \infty, \quad \lim_{x \rightarrow \pi^-} \frac{x^2}{x - \pi} = \frac{\pi^2}{0^-} = -\infty \Rightarrow x = \pi \text{ is V.A.}$$

H.A.

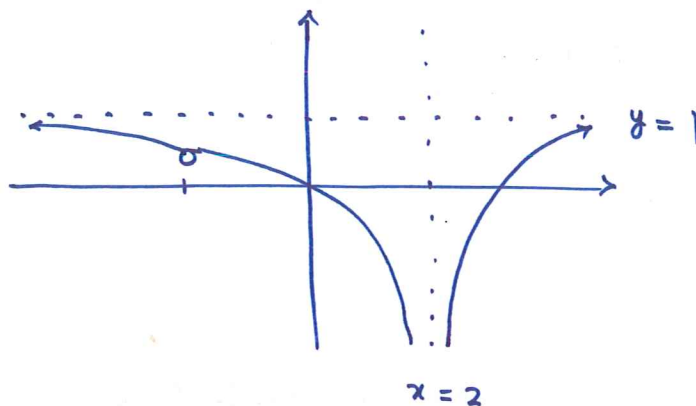
$$\lim_{x \rightarrow \infty} \frac{x^2}{x - \pi} = \lim_{x \rightarrow \infty} \frac{x^2}{x} = \lim_{x \rightarrow \infty} x = \infty \rightsquigarrow \text{NO H.A.}$$

NOT a finite number

3. [3 points] Sketch the graph of a function $f(x)$ satisfying all the following conditions:

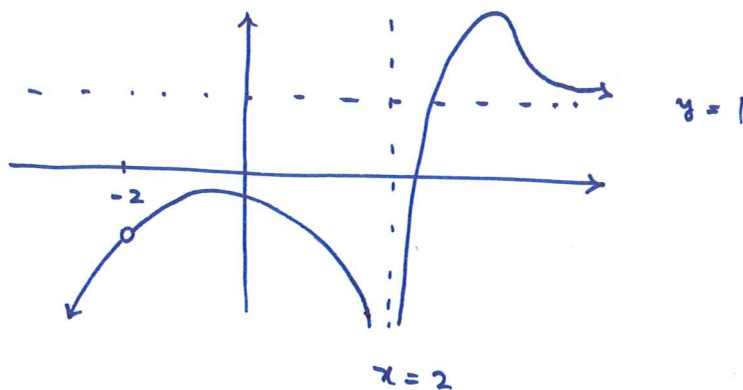
- f is defined everywhere except at $x = 2$ and $x = -2$.
- $\lim_{x \rightarrow 2^+} f(x) = -\infty$ and $\lim_{x \rightarrow 2^-} f(x) = -\infty$
- f has a finite limit at $x = -2$.
- f has a horizontal asymptote at $y = 1$.

Make sure your graph passes the vertical line test to be a function!



* It can be a two sided H.A. or one-sided H.A.

OR



* $x=2$ is the only V.A.
 $x=-2$ is only a hole.