MATH 190, QUIZ 3

Oct 22, 2018
Time: 15 minutes

Show all your work. No calculators, no books/notes are allowed.

Name (please print): ______________________

Student number: Solution ______________________

1. Questions (a)-(e) below all concern the function

$$f(x) = \frac{1}{2x}.$$

(a) [4 points] Use the definition of the derivative (and not any other method) to find $f'(2)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \xrightarrow[x=2]{f'(2)} f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$\boxed{f(x) = \frac{1}{2x} \quad f(2+h) = \frac{1}{2(2+h)}}$$

$$f'(2) = \lim_{h \to 0} \frac{\frac{1}{2(2+h)} - \frac{1}{2.2}}{h} = \frac{0}{0}$$

Simplify

$$= \lim_{h \to 0} \frac{\frac{1}{4+2h} - \frac{1}{4}}{h}$$

* Do NOT forget brackets: $2(2+h)$

Common denominator

$$= \lim_{h \to 0} \frac{\frac{4 - (4+2h)}{(4+2h)4}}{h} = \lim_{h \to 0} \frac{4-4-2h}{(4+2h)4} \cdot \frac{1}{h}$$

* Do NOT forget to distribute (−) in all the term following it.

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{4(4+2h)} \cdot \frac{1}{\cancel{h}} = \frac{-2}{4 \cdot 4} = \boxed{\frac{-1}{8}}$$

(b) [4 points] Use derivative rule(s) to find $f'(2)$.

multiple methods: power rule:

- $y = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2} x^{-1} \Rightarrow y' = \frac{1}{2} \cdot (-1) x^{-1-1} = -\frac{1}{2} \cdot \frac{1}{x^2} \xrightarrow{x=2} f'(2) = -\frac{1}{2} \cdot \frac{1}{2^2} = -\frac{1}{8}$

Quotient Rule $y = \frac{1}{2x} \Rightarrow y' = \frac{(1)' \cdot 2x - 1 \cdot (2x)'}{(2x)^2} = \frac{\cancel{0 \cdot 2x} - 2}{4x^2} = \frac{-2}{4x^2} = \frac{-1}{2x^2}$

(c) [3 points] Find the equation of the tangent line to the graph of $f(x)$ at $x = 2$.

Slope : $m_{tan} = -\frac{1}{8}$

point : $x = 2 \Rightarrow y = f(2) = \frac{1}{2 \cdot 2} = \frac{1}{4} \Rightarrow (2, \frac{1}{4})$

$y - y_0 = m(x - x_0)$

$y - \frac{1}{4} = -\frac{1}{8}(x-2) \Rightarrow y = -\frac{1}{8}(x-2) + \frac{1}{4}$

(d) [2 points] Which option describes the graph of f correctly? Give reasons for your choice.

i. Always increasing

ii. Always decreasing

iii. Increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$

iv. Decreasing on $(0, \infty)$ and increasing on $(-\infty, 0)$

$f'(x) = -\frac{1}{2x^2}$ for all values of x ; $f'(x)$ is negative so slope of the tangent line always negative $\Rightarrow$ function itself is everywhere decreasing.

(e) **(Bonus)** [2 points] Sketch a rough graph of f and its tangent line at $x = 2$.

$y = \frac{1}{2x}$ is similar to $y = \frac{1}{x}$