Welcome to Math 215/255
Ordinary Differential Equations

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- Office: Math Building 229A
- Class location:
  - Mon/Fri: MacMillan (MCML) 166
  - Wed: Pharmaceutical Sciences Building (PHRM) 1101
- Office hours and location:
  – TBD in class on Friday
- Common course page: on Canvas:
  https://canvas.ubc.ca/courses/5932
- Section webpage: lecture notes and any section specific info and announcements here:
  https://blogs.ubc.ca/math215and255s104/

Course Grading

- WebWork 5%
- Homework 10%
- Midterm 1 17.5% (set for Oct 12, in class)
- Midterm 2 17.5% (set for Nov 16, in class)
- Final exam 50%

* NO make-up quiz or exam.

Homework and WebWork

- Webwork and Homework will be assigned in alternate weeks.
- Homework: All homework assignments are posted and are submitted on Canvas. NO late submission.
- There are questions in homework requiring your work in finding numerical solutions or graph plotting in MATLAB. There are TAs to help you with MATLAB throughout the term.
- Please check the details on the course page on Canvas.

Differential Equations (DE)

- The door to understand almost anything in science and engineering.
- Mathematics is the language of science and differential equations are one of the most important parts where this language is used.
- Many natural and physical phenomena are relations involving rates at which things happen; in mathematical terms, relations are equations and rates are derivatives

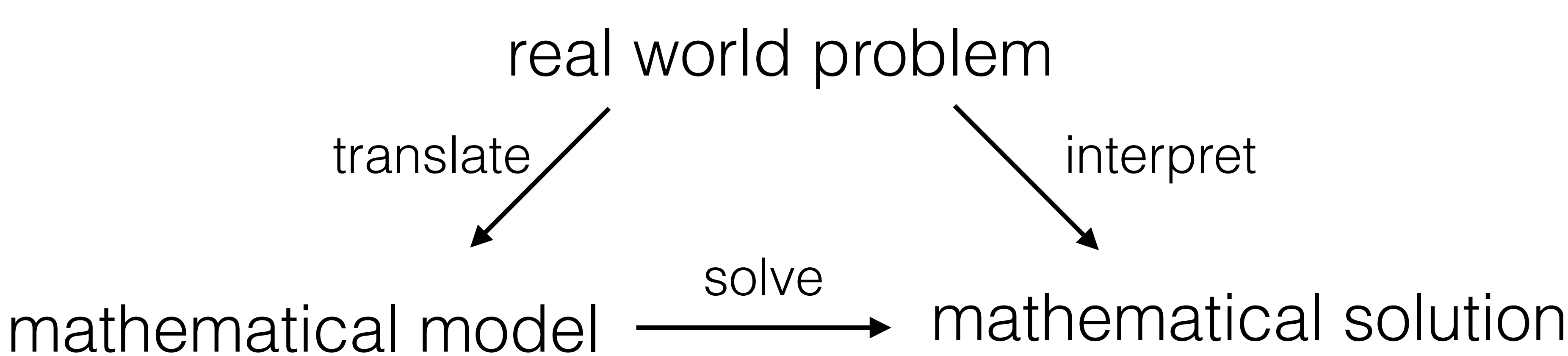
Differential Equations:
Equations containing derivatives

Mathematical Model:

is a differential equation that describes some physical process.

Some examples of these models are:

- Newton's laws
- Fluid mechanics / elasticity….
- Economic and social systems
- Biological population dynamics
- Electrical systems, Stat Mech, …



Solution to a DE

- **Analytic solution:** Find a function that satisfies the equation

Ultimate goal of a mathematician in the field of DE,
but in many cases it is very hard or even impossible! :(

* This is related to my area of research (partial differential equation), look for analytical solution, if we couldn't find any, at least we show there exists one solution (Existence and Uniqueness)

- **Numerical solution:** Use numerical methods and mathematical softwares to know the answer even when we cannot solve analytically.

It is quick and relatively easy in most cases, looking at plots of solutions gives us insight into the behaviour of the solution :)

This week's topic:

- Some examples to see what a differential equation (DE) is
- Classification of DEs: Linear vs nonlinear, 1st order, 2nd order, etc
- Method of separation of variables and Integrating Factor method
- Reading: Lebl 0.2, 1.1, 1.3 ,1.4
- WebWorK 1 is open now. Follow the link on Canvas.
  Due date: Friday, September 14

Differential Equations (DE)

Start with the simple DE:

$$\frac{dy}{dt} = y(t)$$

$y(t)$ = dependent variable

t = independent variable

→ We'd like to solve the equation for y:

* A solution is a function $y(t)$ such that it's equal to its derivative.

<u>Notation</u>: $\frac{dp}{dt} = P(t)$ ↗ solve for P , $\frac{dx}{dt} = x(t)$ ↗ solve for x

* You can choose any letter for indep. or dep. variable.

$$\frac{dy}{dx} = y(x)$$ ↗ solve for y

* Just remember which one is which ⇝ we usually have the derivative of dep. variable with respect to the indep. variable.

• One candidate for the solution: $y(t) = e^t$

$$\frac{dy}{dt} = e^t = y(t)$$

• 2^{nd} candidate: $y = 0$ $\quad \frac{dy}{dt} = 0 = y$ ⇝ trivial solution.

• In general: $y(t) = Ce^t$ is a solution. (C ↙ a constant) → general solution

Verify that it is a solution: $\frac{dy}{dt} = Ce^t = y(t)$ ✓ it satisfies the equation.

Ex1 . Let's add a condition to the equation:

Solve $\left\{ \begin{array}{l} \frac{dy}{dt} = y(t) \\ y(0) = 2 \end{array} \right\}$ → Initial-Value problem

$y(0) = 2$ ← initial condition → Helps us get rid of the Constant C.

General Solution: $y(t) = Ce^t$

$$2 = y(0) = Ce^0 = C \longrightarrow C = 2$$

$$\boxed{y(t) = 2e^t}$$ → particular solution.

(No Constant is in the solution.)

Ex2 . Suppose $u(x) = 5e^{-3x}$ is a solution to the equation $u' + Ku = 0$. Find K?

$$u'(x) = 5 \cdot -3 e^{-3x} = -15 e^{-3x}$$

Since u is a sol'n → $\underbrace{-15e^{-3x}}_{u'} + \underbrace{K \cdot 5 e^{-3x}}_{Ku} = 0$

$$\underbrace{e^{-3x}}_{\text{NEVER } 0}(-15 + 5K) = 0 \Rightarrow -15 + 5K = 0$$

$$\Rightarrow \boxed{K = 3}$$

Ordinary DE (ODE) ⇝ MATH 215/255

$$\frac{dy}{dt} = y(t)$$

Ordinary derivative.

$$\frac{du}{dx} + k\,u(x) = 0$$

⟶ ODE is an equation where the function is only a function of one independent variable.

$y(t)$, $u(x)$, $x(t)$, $f(x)$

Partial Differential Equations (PDE) ⇝ MATH 257/316

Equations involving a function that has multiple independent variables.

$f(t, x, y, z)$

$u(t, x)$

↳ Partial derivatives:

Ex:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \rightsquigarrow \text{Solution: } u(t, x)$$

$\frac{\partial u}{\partial t}$: derivative of u with respect to t

$\frac{\partial^2 u}{\partial x^2}$: derivative of u with respect to x (2^{nd} derivative)

* ∂ : the notation for partial derivative

Order of an ODE : The highest derivative that appears in the equation.

Example :

linear ← $\frac{df}{dt} = f(t)$ → 1^{st} order ODE

Nonlinear ← $\frac{d^4y}{dt^4} = \sin(y) + 2$ → 4^{th} order ODE

(Sin: NOT linear)

Nonlinear ← $y''' + 2e^t y'' + yy' = t^4$

(yy′: NOT linear)

→ 3^{rd} order

Linear. ← $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2 = t^2$ → 2^{nd} order

In this course, we mainly work with 1^{st} and 2^{nd} order ODEs.

Linear vs. NonLinear ODE :

Nice

Suppose we want to solve an ODE for $y(t)$

then, the equation is called linear if

we have only linear functions of $y, y', y'', \dots$

For example; a 1^{st} order linear ODE is

$$a(t) \frac{dy}{dt} + b(t)\, y(t) = c(t)$$

a 2^{nd} order linear ODE

$$a(t) \frac{d^2y}{dt^2} + b(t) \frac{dy}{dt} + c(t)\, y(t) = d(t)$$

* NO e^y ✗
* NO $\sin(y)$, $\cos(y)$ ✗
* NO yy', yy'' ✗
* NO powers except 1: y^2 ✗

Follow the same pattern for higher order ODEs.

Nonlinear ODE : Any other equation that is NOT linear, terms like yy', y^2, $\frac{y''}{y}$, $e^{y'}$, ...

- Hard/impossible to solve
- NO Solution / More than one Solution.