Reminder :

- Office Hours :

  Thursday 4 – 5 pm ⎤
  Friday 4 – 5:30 pm ⎦ → In MATX 1118

- Homework 1 is posted.

  ↳ Online submission (through Canvas)

  Due date : Friday, Sept 21

- Webwork 1 is due tonight. (11:59 pm)

  → For problems 12 & 13 in WW, read the textbook example in Section 1.4
  (My textbook edition : Ex 1.4.2)

Lecture 5
Sept 14

Last Class.

Slope / direction field: important qualitative info about sol'n without finding the sol'n.

autonomous

(1) $\frac{dv}{dt} = g - \frac{K}{m} v$

$\frac{dv}{dt} = 9.8 - \frac{v}{5}$

$V(t)$: dep, indep

$9.8 - \frac{v}{5} = 0 \Rightarrow v = 49$

nonautonomous

(2) $\frac{dy}{dx} = xy$

$y(x)$: dep, indep

Another classification for 1^{st} order ODE:

In general $\frac{dy}{dt} = f(t, y)$

If RHS does NOT depend explicitly on the independent var. (t here), then we call ODE, Autonomous.

$$\boxed{\frac{dy}{dt} = f(y)}$$

Important Info from Autonomous Eqt.

$$\boxed{\frac{dy}{dt} = f(y)} \quad , \quad y(0) = y_0$$

This gives all solutions. (All curves in slope field) ← $\frac{dy}{dt} = f(y)$

$y(0) = y_0$ → With this condition, we will get one particular solution. (One of the curves in slope field)

- Equilibrium Solution:

$y = y^*$ is an equilib sol'n when

$$\frac{dy}{dt} = f(y^*) = 0$$

other names: Critical point or steady states.

- Stability of equilibrium solutions

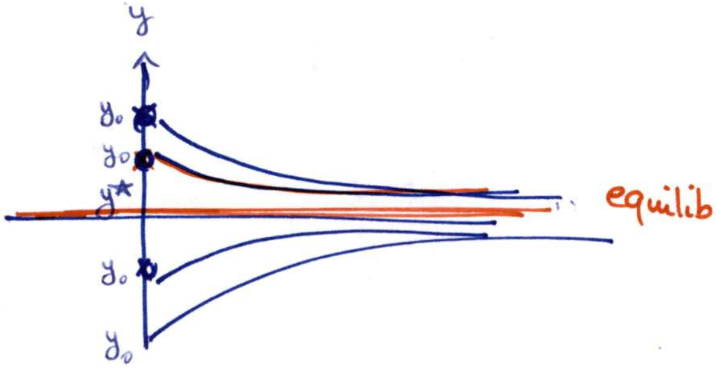
<u>Definition</u>: Equilib sol'n $y = y^*$ is said to be <u>asymptotically stable</u> when

solutions that start near $y = y^*$ all move toward y^* as t increases. i.e.

If y_0 is close to y^* then $y(t) \longrightarrow y^*$ as $t \to \infty$.

y_0: initial condition

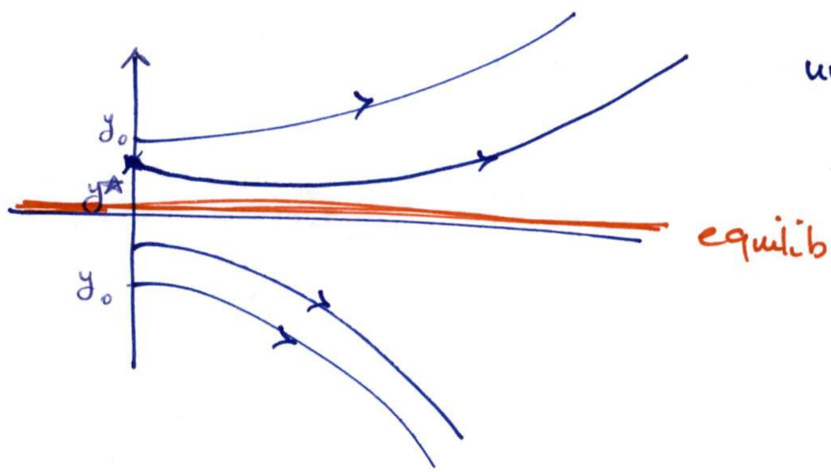
"is close to y^*": This has a rigorous mathematical formulation



* The solution curves near an asymptotically stable equilibrium look like this figure

→ **Asymptotically unstable equilib sol'n :**

All sol'ns start near $y = y^*$, but as t increase they move away from $y = y^*$



unstable.

(*) For a stable solution, as we see, small perturbations of the equilibrium solution are all damped out, however, this is NOT the case for an unstable equilibrium solution.

Ex.

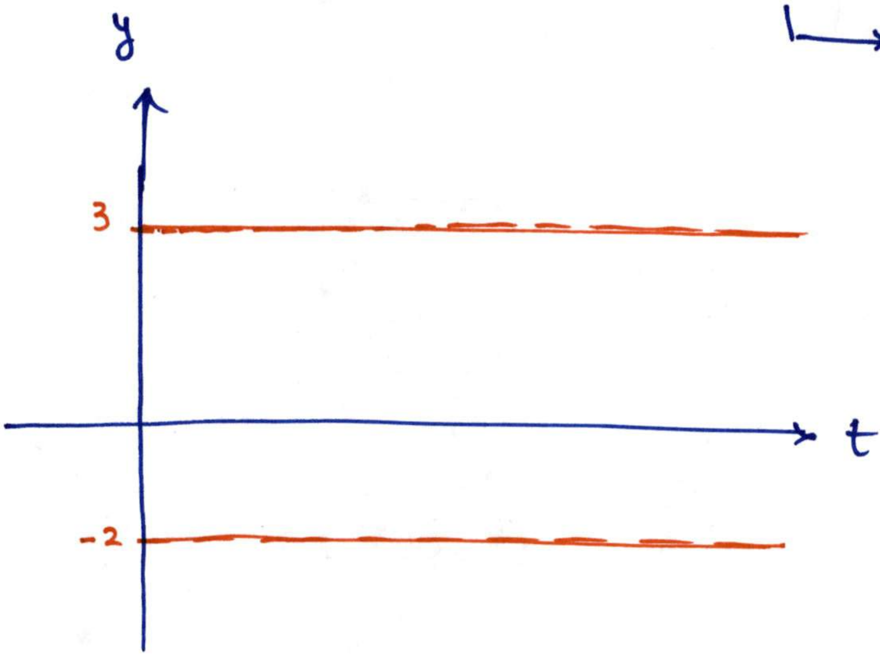
Find and Classify equilib. sol'n.

(1) $\frac{dy}{dt} = \underbrace{y^2 - y - 6}_{f(y)}$ autonomous 1^{st} order

$$\frac{dy}{dt} = f(y) = y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0 \quad \to \quad y = 3, \quad y = -2$$

steady states.



Next class: Stable / unstable ?