II

FUNCTION IS A RULE WHICH ASSIGNS TO EACH REAL NUMBER $x$ IN ITS DOMAIN A UNIQUE REAL NUMBER $f(x)$

EX

$\underbrace{\text{RULE WHICH ASSIGNS TO EACH}}$ # ITS SQUARE

" # $x$ THE # $\dfrac{x^3+3x+5}{x^2+1}$

" # $y \neq 1$ OR $-1$ THE # $\dfrac{y^3+3y+5}{y^2-1}$

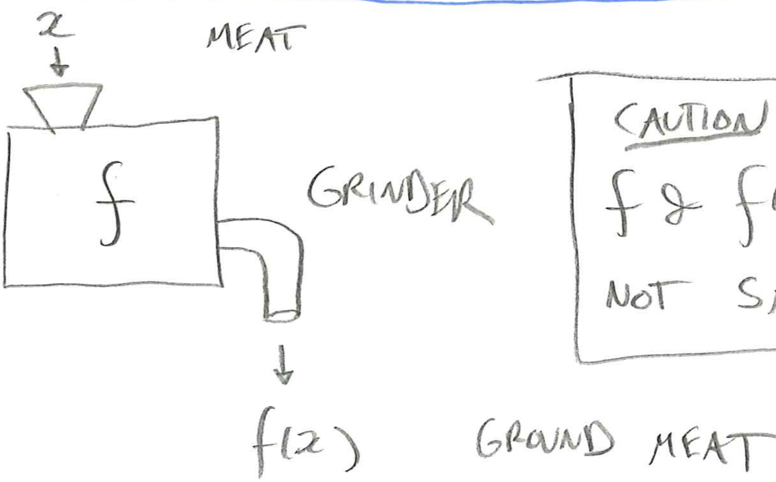
" # THE VALUE '1', & PIECE WISE

$\boxed{\text{WHAT IS A RATIONAL NUMBER?}}$

IS RULE WHICH ASSIGNS TO EACH RATIONAL # THE VALUE '1' & EACH IRRATIONAL # THE VALUE '0' A FUNCTION?

FORMULAS

## THE MEAT GRINDER PERSPECTIVE




**CAUTION**

$f$ & $f(x)$ NOT SAME

## THE GRAPHICAL PERSPECTIVE

THINK OF (INPUT, OUTPUT) AS AN ORDERED PAIR $(x, f(x))$

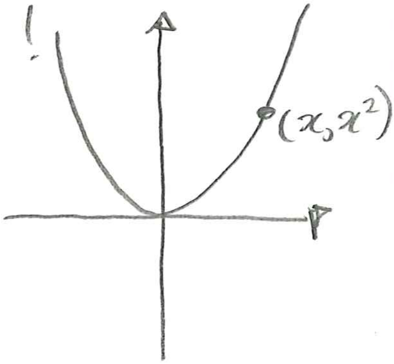
THINK OF THE FUNCTION $f$ AS THE COLLECTION OF ORDERED PAIRS SPECIFYING ITS VALUES

$\boxed{\text{EX}}$ $f(x) = x^2$ WOULD BE REPRESENTED BY ALL PAIRS $(x, x^2)$ WHERE $x$ IS ANY REAL #

WHY IS THIS THE GRAPHICAL PERSPECTIVE?

IV

BECAUSE THE COLLECTION OF THESE PAIRS CAN BE GRAPHED!




WE SEE THE WHOLE FUNCTION AT ONCE.

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EX| IF DOMAIN IS FINITE WE CAN LIST ALL THE PAIRS

DOMAIN$(f) = 1$ & $2$

$f(1)=3$, $f(2)=4$

THEN $f$ IS COLLECTION OF PAIRS $(1,3)$ & $(2,4)$

A FUNCTION IS A BUNCH OF INPUTS WITH UNIQUE OUTPUTS

CAN WE DEFINE A FUNCTION AS A BUNCH OF ORDERED PAIRS?

V

A FUNCTION IS A COLLECTION OF ORDERED PAIRS OF NUMBERS (INP/OUT) WITH THE SPECIAL PROPERTY THAT IF $(x,y)$ & $(x,z)$ ARE IN THE COLLECTION THEN $y=z$

WHAT DOES THE SPECIAL PROPERTY CORRESPOND TO IN A GRAPH?

VERTICAL LINE TEST!

TWO POINTS OF VIEW ON SAME THING, BOTH HAVE ADVANTAGES & DISADVANTAGES

WHAT DO YOU THINK THEY ARE?

VI

- THINK OF LECTURES AS DISCUSSION
- YOU WILL BE EXPECTED TO DO SOME PRE-READING (PARTICIPATION MARKS)
- I WILL NOT REPEAT MATERIAL AS IT APPEARS IN TEXT
- YOU WILL BE EXPECTED TO COMPARE AND CONTRAST

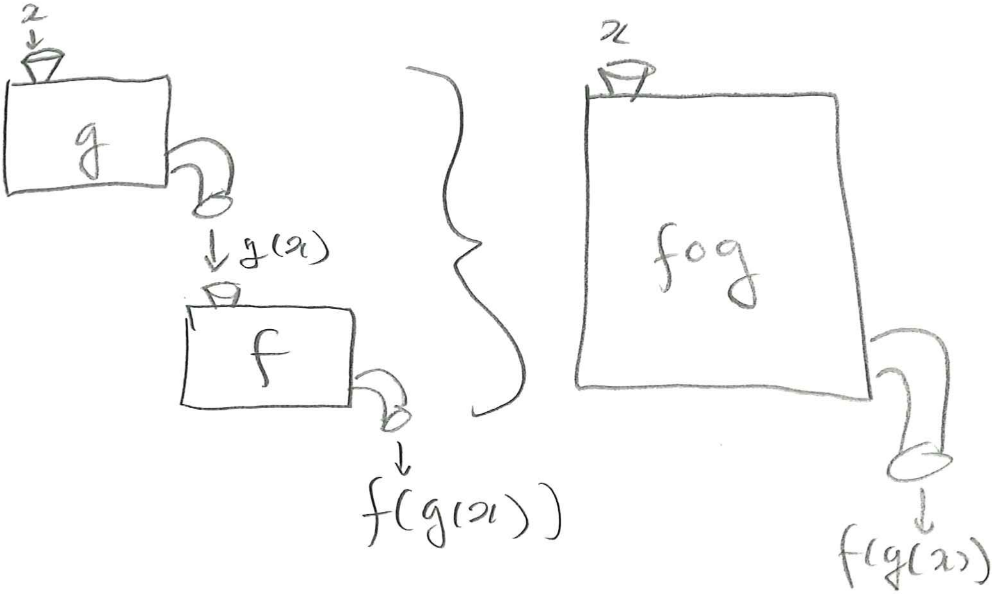
<u>WARNING</u>: YOU WILL BE REQUIRED TO THINK

- STYLE WILL VARY FROM LECTURE TO LECTURE
- OFFICE HOURS / TUTORIAL

# ADVANTAGE MEAT GRINDER

VII

**FUNCTION COMPOSITION** MEANS DOING THINGS ONE AFTER ANOTHER




**NOTATION** $\boxed{(f \circ g)(x) := f(g(x))}$

**NATURAL QUESTION**

CAN WE ALWAYS "UNDO" THE EFFECT OF A FUNCTION, i.e., GIVEN $f$ IS THERE $g$ SUCH THAT

$$f(g(y)) = y \quad \text{AND} \quad g(f(x)) = x$$

YES / NO? HOW?

IDEA REVERSE INPUTS AND OUTPUTS OF $f$ TO OBTAIN A $g$??

EX | LET $f$ BE A FUNCTION WHOSE DOMAIN IS $\{1, 2\}$ & RANGE IS $\{3, 4\}$ DEFINED BY PAIRS $(1, 3)$ & $(2, 4)$

IF $g$ IS DEFINED TO HAVE DOMAIN $\{3, 4\}$ & RANGE $\{1, 2\}$ & PAIRS $(3, 1)$ & $(4, 2)$

THEN $f(g(3)) = f(1) = 3$

$g(f(1)) = g(3) = 1$ etc.

IT SEEMS TO WORK...

PROBLEM: $g$ DEFINED BY REVERSING PAIRS MAY NOT BE A FUNCTION.

(INPUTS, OUTPUTS)

EX) LET $f$ HAVE DOMAIN $\{1,2\}$ IX

& RANGE $\{3\}$ BE DEFINED

BY $(1,3)$ & $(2,3)$

THEN THE MAP $g$ WITH DOMAIN

$\{3\}$ & RANGE $\{1,2\}$ DEFINED

AS $(3,1)$ & $(3,2)$ MAKES NO

SENSE (NOT ! OUTPUT!)

WHAT IS DIFFERENT HERE?

A FUNCTION $f$ IS $\boxed{\text{ONE-TO-ONE}}$ IF

$f(a) \neq f(b)$ WHENEVER $a \neq b$

WHAT DOES THIS MEAN GRAPHICALLY?

HORIZONTAL LINE TEST !! DRAW

THM IF $f$ IS ONE-TO-ONE FUNCTION

ON DOMAIN $D$ WITH RANGE $R$ THEN $\exists!$

INVERSE $g$ WITH DOMAIN $R$ & RANGE $D$

$g(f(x)) = x$ & $f(g(y)) = y$

X

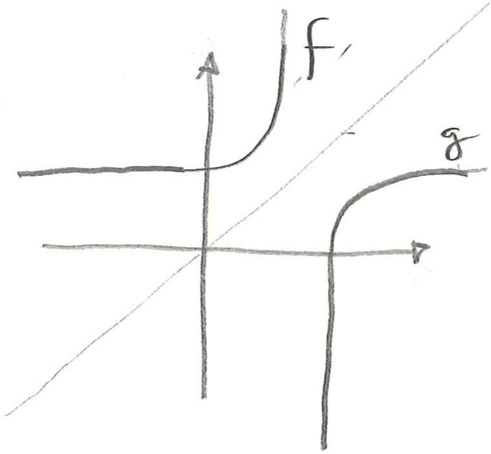
HOW DO WE GET $g$?

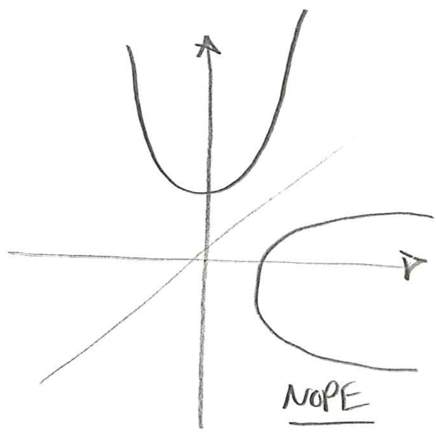
REVERSE THE PAIRS DEFINING $f$

WHAT HAPPENS GRAPHICALLY WHEN WE REVERSE THE PAIRS?

WE REFLECT PLANE ALONG THE DIAGONAL

EX




PREREAD SECTION 1.3

(OR AT LEAST LOOK AT PICS)

NEXT CLASS WE'LL TALK ABOUT MAKING MONEY...