MATH 104 SECTION 108

MATH.UBC.CA/~MBERGERON/MATH104.HTML

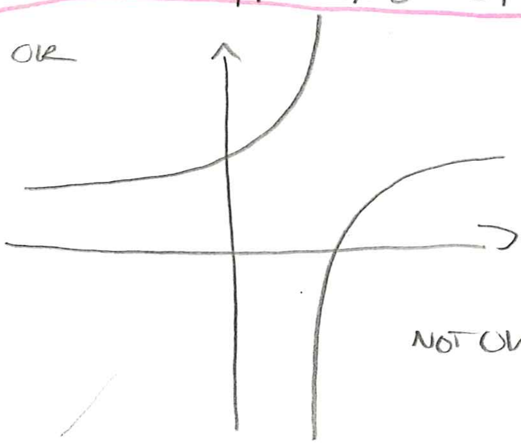
I

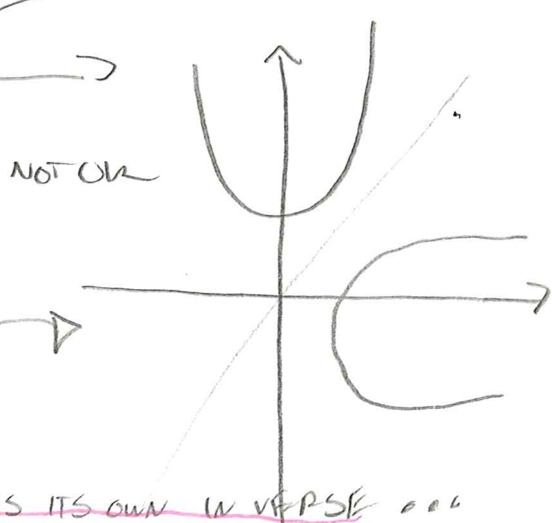
A FUNCTION IS ONE-TO-ONE IF $f(a) \neq f(b)$ WHENEVER $a \neq b$

THM IF f IS ONE-TO-ONE ON A DOMAIN D THEN IT HAS A UNIQUE INVERSE

HOW DO WE OBTAIN THIS INVERSE?

WHAT IS HAPPENING GRAPHICALLY?





WHERE IS IT INVERTIBLE?

FIND A FUNCTION THAT IS ITS OWN INVERSE...

SOME "EASY" FUNCTIONS

II

$f(x) =$ SIMPLE FORMULA FOR x

EASIEST ONES TO DEAL WITH ARE

POLYNOMIAL FUNCTIONS

$$f(x) = a_0 + a_1 x^1 + a_2 x^2 + \cdots + a_n x^n$$

FOR CONSTANTS $a_0, \ldots, a_n$

WHAT IS DOMAIN OF f? IMAGE?

ALL REAL NUMBERS (NOTHING TO WORRY ABOUT)

EX $f(x) = \dfrac{3x+1}{x^2-1}$ DOMAIN EXCLUDES ± 1

SOME HARDER FUNCTIONS MIGHT INVOLVE EXPONENTIATING (DECAY PROBLEMS)

CAN WE MAKE SENSE OF THE FORMULA $f(x) = 2^x$?

IS IT A FUNCTION? WHAT IS DOMAIN?

WHAT IS 2^0, 2^{-1}, 2^{-n}, $2^{\frac{1}{n}}$? III

WHAT ABOUT $2^{\sqrt{2}}$ OR 2^{π}??

WOULD LIKE DOMAIN OF 2^x TO BE $\mathbb{R}$...

A MAGICAL NUMBER SAVES DAY

$\boxed{e = 2.7182818\ldots}$ LIKE π

HAS MYSTERIOUS PROPERTY

$$\boxed{e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots + \frac{1}{k!}x^k + \cdots}$$

COMPOUND INTEREST

INFINITE POLYNOMIAL

WHAT DOES THAT EVEN MEAN

(PRETEND IT IS JUST A FINITE ONE UP TO k = HUGE NUMBER)

THIS IS A TAYLOR SERIES (MORE LATER)

STOP & COMPUTE FOR ME

WHY IS THIS FANTASTIC NEWS?

DOMAIN OF $f(x) = e^x$ IS ALL $\mathbb{R}$.

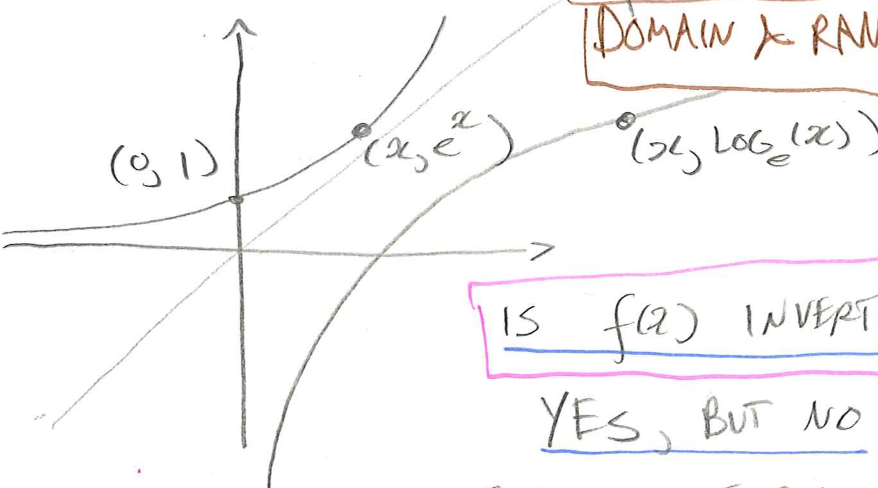
$f(x) = e^x$ SATISFIES PROPERTIES YOU EXPECT (IV)

CRUCIAL ONE $f(x_1 + x_2) = e^{x_1 + x_2} = e^{x_1} e^{x_2} = f(x_1) f(x_2)$

TURNS ADDITIONS INTO MULTIPLICATIONS

WHAT DOES GRAPH OF $f(x) = e^x$?

DOMAIN & RANGE?



IS $f(x)$ INVERTIBLE?

YES, BUT NO EXPLICIT FORMULA

SINCE FUNCTION EXTREMELY IMPORTANT GIVE INVERSE A SPECIAL NAME:

WE DENOTE BY $g(y) = \log_e(y) = \ln(y)$ THE INVERSE OF $f(x) = e^x$

$f(g(y)) = y$ & $g(f(x)) = x$

$e^{\log_e(y)} = y$ & $\log_e(e^x) = x$

CAN WE DEDUCE PROPERTIES OF $g(y) = \mathrm{LN}(y)$ FROM $f(x) = e^x$?

V

$$\mathrm{LN}(a) + \mathrm{LN}(b) = \mathrm{LN}\left(e^{\mathrm{LN}(a) + \mathrm{LN}(b)}\right)$$

$$= \mathrm{LN}\left(e^{\mathrm{LN}(a)} e^{\mathrm{LN}(b)}\right) = \mathrm{LN}(a \cdot b)$$

THIS IS THE "DO NOTHING" TRICK

APPLYING A FUNCTION ($f \circ f^{-1}$, $\mathrm{LN} \circ e$)

WHICH ACTS AS THE IDENTITY

IT WILL COME UP OFTEN

CAN YOU USE A SIMILAR APPROACH TO FIND A FORMULA FOR $b\,\mathrm{LN}(a)$?

$$b\,\mathrm{LN}(a) = \mathrm{LN}\left(e^{b\,\mathrm{LN}(a)}\right) = \mathrm{LN}\left(\left(e^{\mathrm{LN}(a)}\right)^b\right)$$

$$= \mathrm{LN}(a^b)$$

THESE ARE THE KIND OF MANIPULATIONS WHICH ALLOW YOU TO SOLVE EQUATIONS

HAVE WE MADE PROGRESS TO UNDERSTAND WHETHER 2^{π} MAKES SENSE? (VI)

KNOW: e^x = INFINITE POLYNOMIAL

$\Rightarrow f(x) = e^x$ HAS DOMAIN $\mathbb{R}$

KNOW: $g(y) = \ln(y)$ IS INVERSE OF $f(x) = e^x$

$\Rightarrow e^{\ln(y)} = y$ "DO NOTHING" TRICK

KNOW $\ln(a^b) = b \ln(a)$

CAN WE USE THIS TO WRITE A FORMULA FOR $h(x) = 2^x$ OR $h(x) = b^x$?

FOR ANY # $b \geq 0$ ~~[illegible]~~

$$\boxed{h(x) = b^x = e^{\ln(b^x)} = e^{x \ln(b)}}$$

~~[illegible]~~

THIS IS JUST $e^{\text{SOME POWER}}$

SO YES WE NOW HAVE FULL DOMAIN