

MATH 104 - SECTION 108

(I)

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↳ HOW TO SUCCEED IN COURSE (LEARNING GOALS)

↳ ADDITIONAL NOTES FOR TODAY (NOT IN BOOK)

→ PIAZZA: MATH 104/08 ←

MATHEMATICS IS A LANGUAGE USED TO DESCRIBE RELATIONSHIPS

FOR US THESE RELATIONSHIPS
ARISE AS FUNCTIONS.

TO SEE HOW THIS WORKS LET'S
DESCRIBE AN (UNREALISTICALLY)
SIMPLISTIC BUSINESS PROBLEM

HOW MANY ARE IN BUSINESS?

DON'T WORRY, BUSINESS IS

EASIER THAN MATH, MAY BE

THAT'S WHY YOU ARE HERE

RUN A BUSINESS

(II)

SELL STUFF (EG.)

Q WHAT QUANTITIES SHOULD YOU CARE ABOUT?

PRICE

P SINGLE UNIT

DEMAND

$q()$ QUANTITY

COST

PRODUCTION COST $c()$

REVENUE

MONEY MADE BY SELLING $R()$

PROFIT

MONEY REALLY MADE $P()$

Q HOW ARE THESE GUYS RELATED TO EACH OTHER?

WHICH ONES CAN WE CONTROL?

P , $q(P)$, $c(q)$, $R(q) = P \cdot q$, $P(q) = R(q) - c(q)$

HOW CAN WE FIGURE THEM OUT?

HOW ARE PRICE & DEMAND RELATED?

III

INCREASE $p \Rightarrow q(p)$ DECREASES

CAN GET PRETTY COMPLICATED, *TALK SWINE APPLE NOT INTUITIVE*

MATH PRINCIPLE: ALWAYS TRY TO

UNDERSTAND SIMPLEST CASE FIRST

Q: WHAT ARE EASIEST FUNCTIONS?

LINEAR

MODEL PRICE / DEMAND RELATIONSHIP BY

A LINEAR FUNCTION: $q(p) = ap + b$

CONSTANTS a & b

WHAT DOES PRODUCTION COST LOOK LIKE?

FIXED COSTS VS VARIABLE COSTS

WHAT DOES REVENUE LOOK LIKE?

$$R(q) = p \cdot q$$

WHAT HAPPENS WHEN REVENUE = COSTS?

BREAK EVEN

YOU HAVE A BUSINESS WHICH SELLS (TV)
THINGS PRODUCED IN A FACTORY
YOU HIRE AN ECONOMIST WHICH TELLS YOU:

- WHEN THINGS ARE PRICED AT 200\$/UNIT
THEN WEEKLY DEMAND IS 5000 UNITS
- EVERY INCREASE OF PRICE BY A DOLLAR
RESULTS IN A DECREASE OF DEMAND
BY 50 UNITS / WEEK
- FIXED WEEKLY PRODUCTION COST IS
100,000\$ & VARIABLE PRODUCTION
COSTS ARE 75\$/UNIT

Q: WHAT IS THE LINEAR
DEMAND EQUATION FOR
THINGS YOU ARE SELLING

(VI)

EXPECT? $q(p) = ap + b$

NEED TO FIND? a & b VALUES

WHAT INFO CAN WE USE?

$$\underline{5000 = 200a + b}$$

OR $\left\{ \begin{array}{l} +1 \rightarrow -50 \Rightarrow \text{SLOPE OF LINE } -50 \\ 4950 = 201a + b \end{array} \right.$

$$\underline{4950 = 201a + b}$$

HOW CAN WE USE THIS INFO?

① $\underline{5000 = 200a + b}$

② $\underline{4950 = 201a + b}$

SOLVE FOR ONE

OF THE
VARIABLES

①-② GET $\boxed{-50 = a}$

SUBSTITUTE BACK INTO ①

$$\underline{5000 = -10000 + b \Rightarrow b = 15000}$$

$$q = -50p + 15000$$

$$p = \frac{-q}{50} + 300$$

→ SOMETIMES
USEFUL
→ PERSPPECTIVE

WHAT IS COST?

$$C(q) = 100\,000 - 75q$$

VI

WHAT IS WEEKLY REVENUE?

$$R(q) = p \cdot q = \left(300 - \frac{q}{50}\right) \cdot q$$

WHAT IS WEEKLY PROFIT?

$$P(q) = R(q) - C(q)$$

$$= \left(300 - \frac{q}{50}\right)q - (100\,000 - 75q)$$

$$= 300q - \frac{q^2}{50} - 100\,000 + 75q$$

$$P(q) = -\frac{q^2}{50} + 225q - 100\,000$$

SKETCH
 $6 \cdot 10^5$



WHERE DO YOU MAKE MONEY?

VII

HOW DO WE MAXIMIZE PROFIT?

FIND THE PEAK OF THE PARABOLA

IF IN FORM LIKE: $y = ax^2 + bx + c$

KNOW MAXIMUM IS AT $x = \frac{-b}{2a}$

WHAT DO WE GET?

$$P(q) = -\frac{q^2}{50} + 225q - 100000$$

$$\text{MAX AT } q = \frac{-225}{(-2/50)} = (-225) \cdot \frac{50}{-2}$$

$$= \frac{225 \cdot 50}{2} = 5625$$

MAX PROFIT IS $P(5625)$

HW: WHAT ARE BREAK EVEN POINTS?
WHAT ARE THEY IN OUR GRAPH?

READ EXTRA NOTE 'A BASIC BUSINESS'
PROBLEM

How REALISTIC WAS THAT?

VIII

NOT AT ALL

RELATION SHIPS MUCH MORE COMPLICATED

⇒ NEED TO FIND MAX OF COMPLICATED FUNCTIONS

MAIN IDEA OF CALCULUS: USE

EASY FUNCTIONS (LINEAR, POLYNOMIAL)

TO UNDERSTAND COMPLICATED FUNCTIONS,

↳ EG. MAXIMIZE, *how?*

START EASY: HOW CAN WE

UNDERSTAND GRAPH OF A NICE FUNCTION

AT A POINT BY USING LINES?

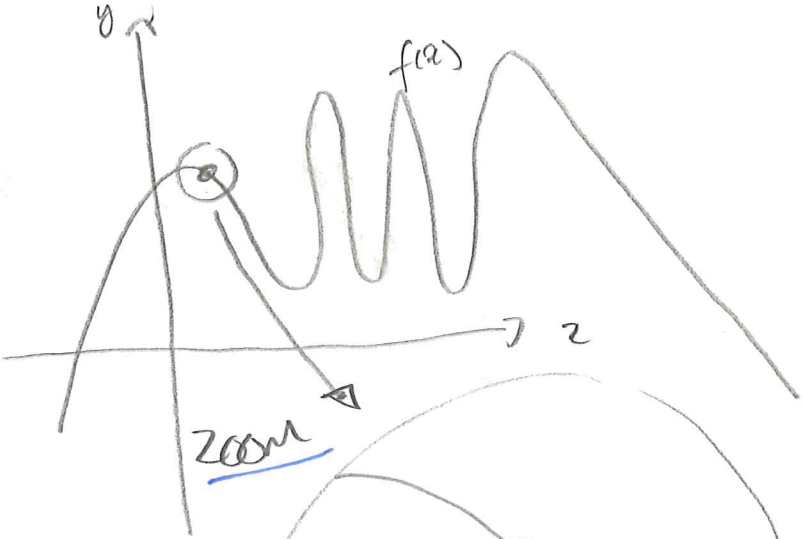
TAKE THE LINE THAT LOOKS

MOST LIKE THE GRAPH AT THAT POINT.

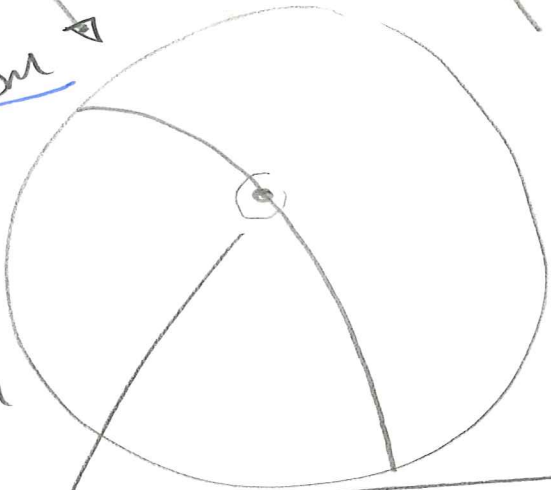
HOW CAN I FIND THIS LINE?

ZOOM IN

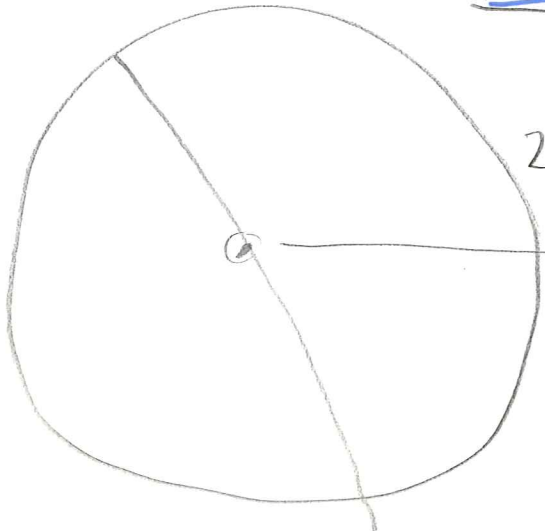
IX



ZOOM



WIKIPEDIA TANGENT

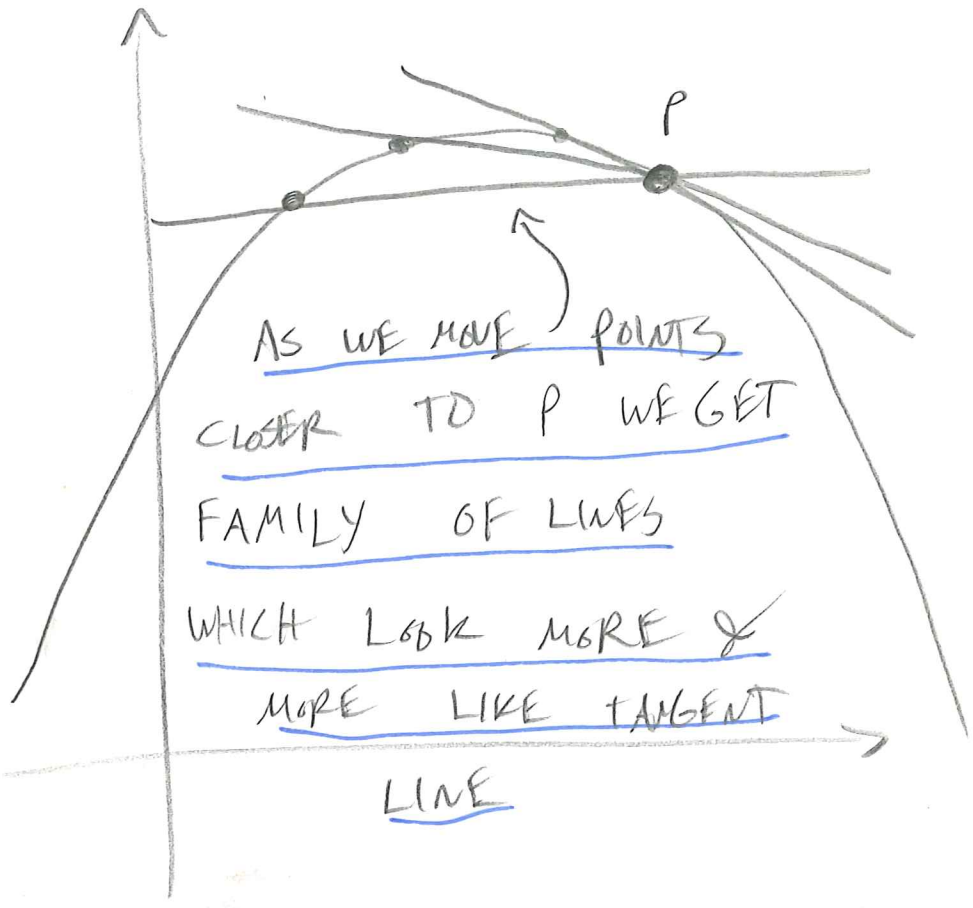


zoom



APPROXIMATE BY SECANTS

(X)



Q WHERE HAVE WE SEEN?
THIS IDEA BEFORE?

CAREFULLY READ SECTION 2.2

→ TRY & START WORK