I

WWW.MATH.UBC.CA/~MBERGERON/MATH104.HTML

↳ WEBWORK DUE WEDNESDAYS 8 AM

↳ USE PIAZZA TO DISCUSS APPROACHES (DON'T POST ANSWERS)

↳ WORK IN GROUPS

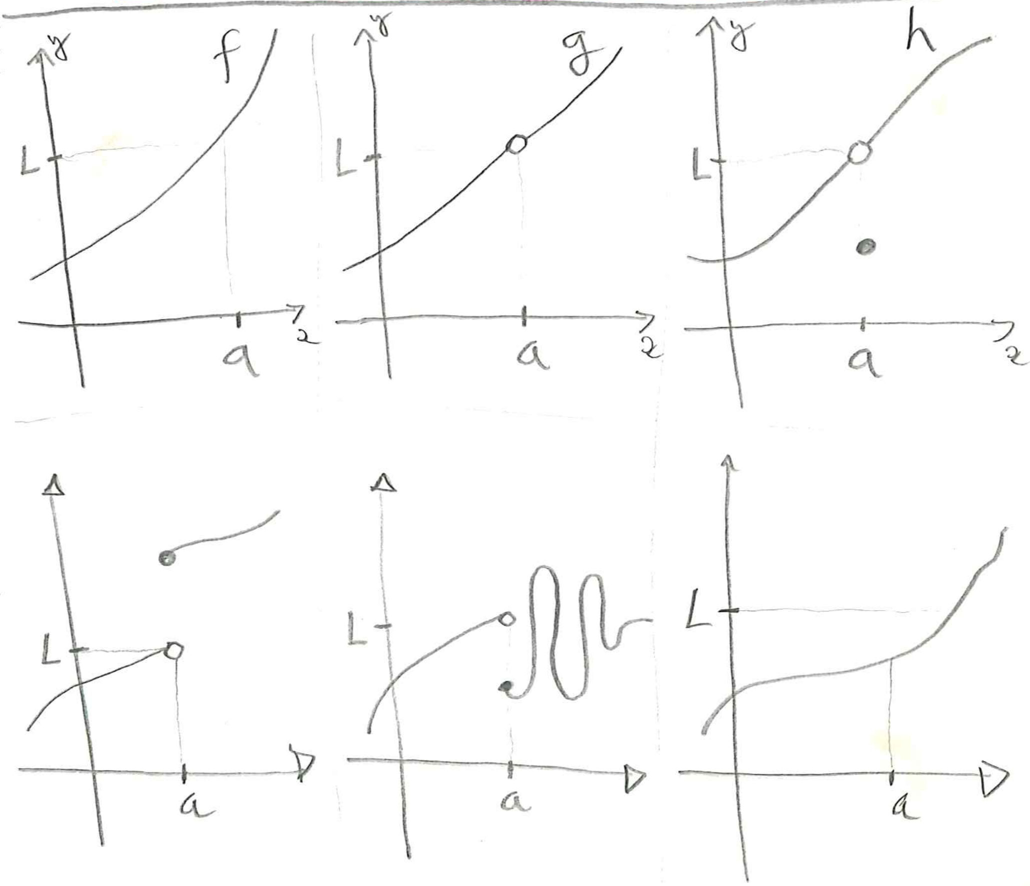
(YOU HAVEN'T UNDERSTOOD A PROBLEM UNTIL YOU HAVE SUCCESSFULLY EXPLAINED IT TO SOMEONE)

↳ WHERE SHOULD YOU WORK?

MATH LEARNING CENTRE !!!

- SOCRATES
- QUESTIONS NOT ANSWERS
- FIND TAS YOU LIKE

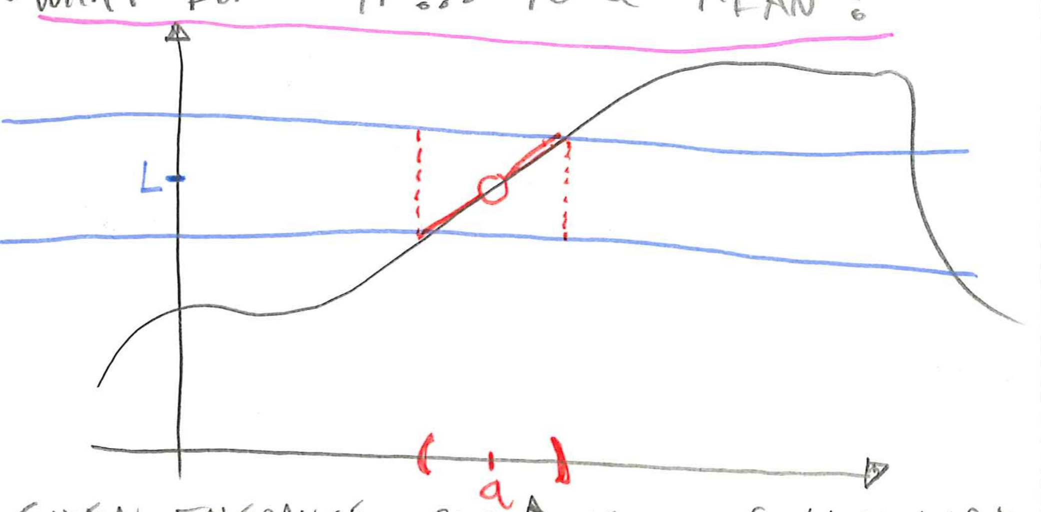
↳ WHAT TO EXPECT???



II

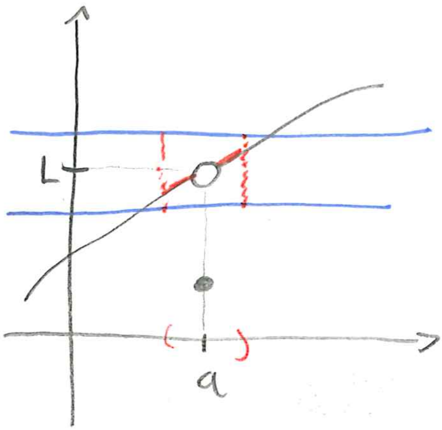
THE FUNCTION f APPROACHES THE LIMIT L NEAR A POINT a (IN SYMBOLS $\lim_{x \to a} f(x) = L$) IF WE CAN MAKE $f(x)$ AS CLOSE AS WE LIKE TO L BY REQUIRING x TO BE SUFFICIENTLY CLOSE, BUT UNEQUAL, TO a.

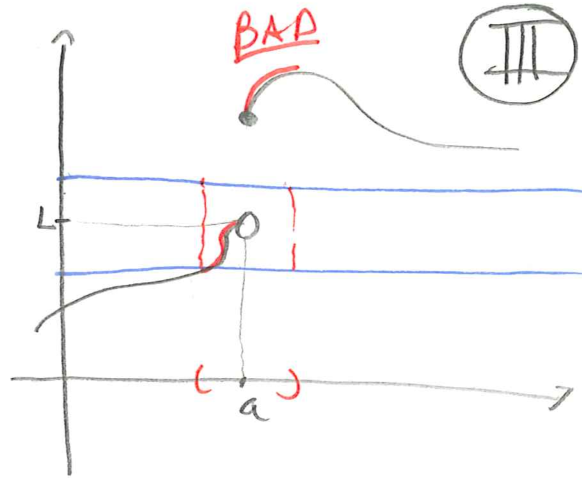
- WHAT DOES IT MEAN FOR #'S TO BE CLOSE?
- WHAT DOES IT MEAN TO MAKE $f(x)$ CLOSE TO L?
- WHAT DOES IT MEAN TO REQUIRE x TO BE CLOSE, BUT UNEQUAL, TO a?
- WHAT DOES IF... TO a MEAN?

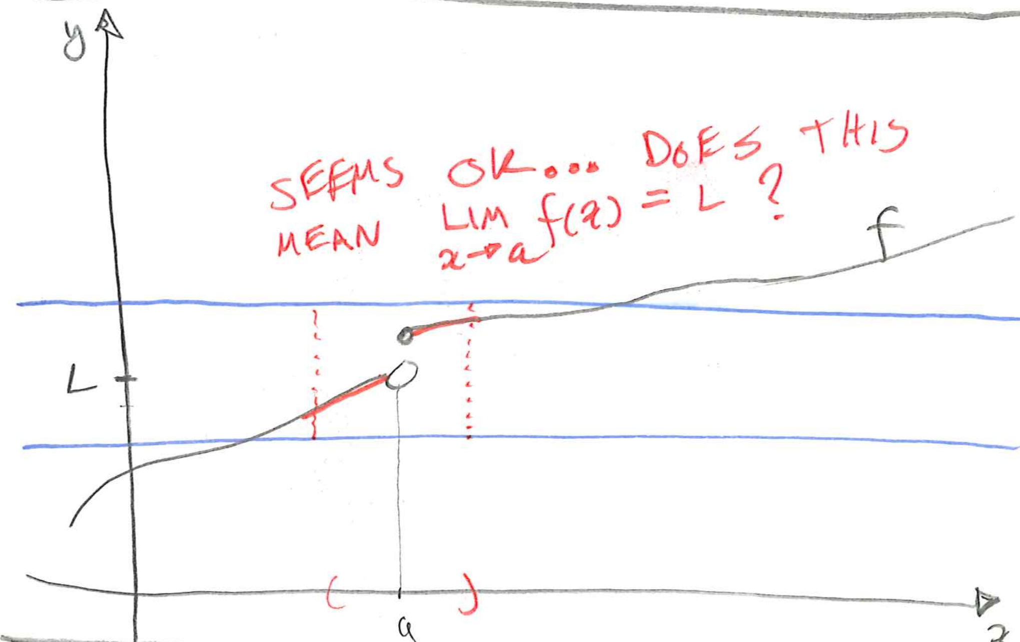


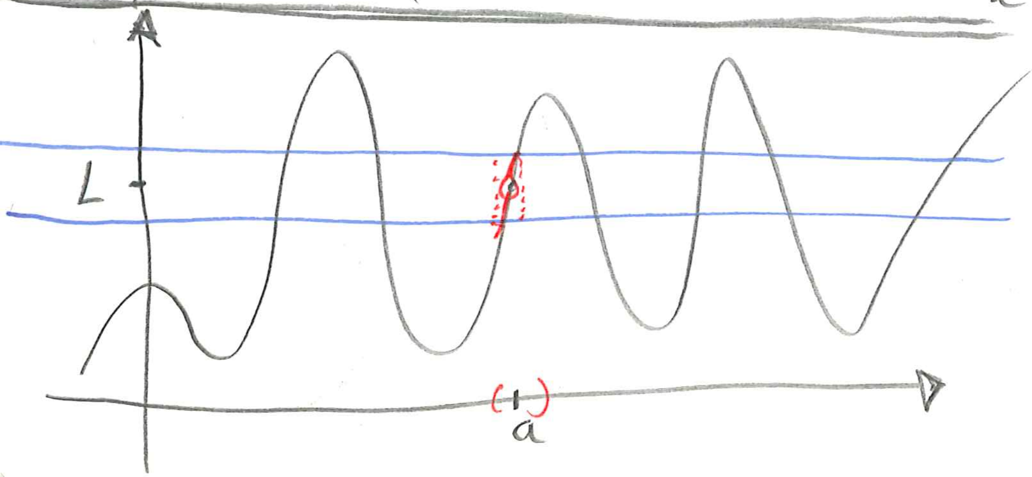
GIVEN TOLERANCE FIND δ SMALL SO f LANDS IN BOX

III









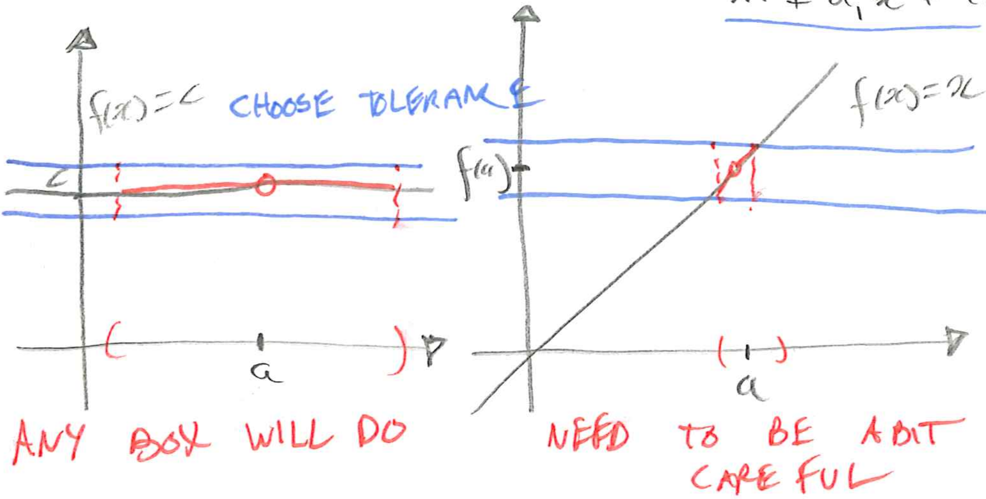
IF THE GRAPH OF A FUNCTION DOESN'T HAVE JUMPS, LIMITS ARE WHAT YOU EXPECT, OTHERWISE BEWARE **IV**

WHAT FUNCTIONS DO YOU KNOW WITH NICE GRAPHS?

CONSTANT FUNCTION $f(x) = c$

LINEAR FUNCTION $f(x) = ax + b$

POLYNOMIAL FUNCTIONS $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0$



FOR THESE NICE FUNCTIONS WE HAVE

$$\boxed{\lim_{x \to a} f(x) = f(a)}$$

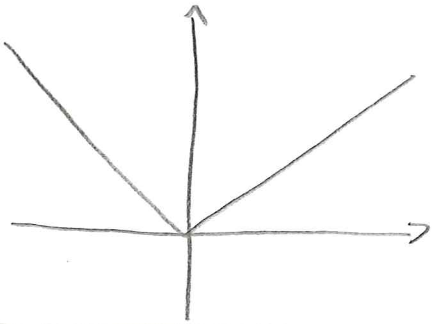
WE CALL THESE GUYS CONTINUOUS FUNCTIONS

WHAT IS THE ABSOLUTE VALUE OF #? (V)

IS IT A FUNCTION?

$f(x) = |x|$ WHERE $|x| := \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x < 0 \end{cases}$

WHAT DOES ITS GRAPH LOOK LIKE?



WHAT IS

$\lim_{x \to a} |x| = |a|$?

LET'S USE THIS FUNCTION TO MAKE DEF$^{\underline{n}}$ OF LIMIT MORE PRECISE!

MAKE $f(x)$ CLOSE TO L MEANS ?

$|f(x) - L|$ SMALL #

MAKE x CLOSE TO a MEANS

$|x - a|$ SMALL #

f APPROACHES LIMIT L NEAR a IF WE CAN MAKE $|f(x) - L|$ AS SMALL AS WE LIKE BY REQUIRING THAT $|x - a|$ IS SUFFICIENTLY SMALL AND $x \neq a$.

THESE ARE THE BOXES WE'VE BEEN DRAWING !!!

VI

WHAT'S THE ADVANTAGE OF DOING THIS ?

DON'T NEED TO DRAW PICTURES !

2 EASY EXAMPLES, THEN TACKLE INSANE FUNCTION

$\boxed{f(x) = x}$, $\boxed{\text{WHY IS } \lim_{x \to a} f(x) = a}$?

I GIVE YOU SMALL TOLERANCE, $\frac{1}{100}$

NOW YOU NEED TO FIND A WAY TO MAKE x CLOSE ENOUGH TO a, i.e. $|x-a|$ SMALL ENOUGH TO MAKE SURE $|f(x)-a| < \frac{1}{100}$

WELL SINCE $f(x) = x$

IF I MAKE $|x-a| < \frac{1}{100}$

THEN THIS IS THE SAME AS

$$|f(x)-a| = |x-a| < \frac{1}{100}$$

i.e. $|f(x)-a| < \frac{1}{100}$ AS DESIRED

THIS WORKS FOR ANY TOLERANCE SO DONE

INSANE FUNCTION

VII

$\boxed{f(x) = \sin(x)}$ EVERYBODY KNOWS

$\boxed{f(x) = \sin\left(\frac{1}{x}\right)}$ DOMAIN? RANGE?

WHAT DOES IT LOOK LIKE?

WHAT IS $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$?

IS FUN DEFINED THERE?

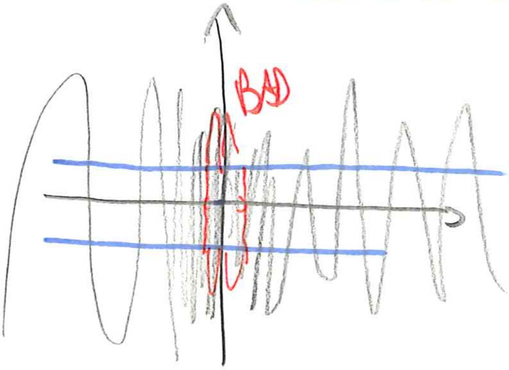
WHAT ABOUT $g(x) = x \sin\left(\frac{1}{x}\right)$

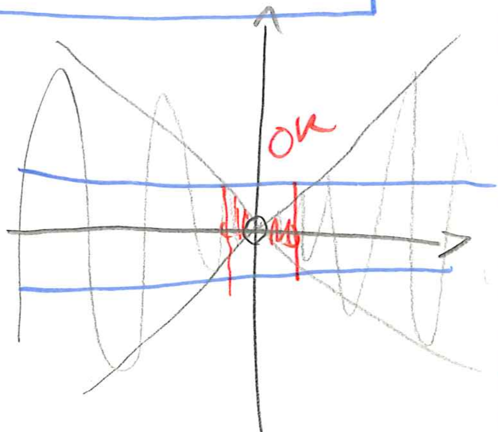
DOMAIN?

WHAT DOES IT LOOK LIKE?

WHAT IS $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$?

GUESS ZERO, HOW TO CHECK?





TO SEE $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$ VIII

GIVEN TOLERANCE, SAY $\frac{1}{10}$,

NEED TO MAKE $|x-0| = |x|$

SMALL ENOUGH THAT $\left|x \sin\left(\frac{1}{x}\right) - 0\right| = \left|x \sin\left(\frac{1}{x}\right)\right|$

IS SMALLER THAN $\frac{1}{10}$.

USE SPECIAL PROPERTY OF SIN

↳ $\left|\sin\left(\frac{1}{x}\right)\right| \leq 1$ (EXCEPT $x=0$)

MEANS

$\left|x \sin\left(\frac{1}{x}\right)\right| \leq |x| < \frac{1}{100}$

SO IF FORCE

WE GET $\left|x \sin\left(\frac{1}{x}\right)\right| < \frac{1}{100} < \frac{1}{10}$

SINCE THIS WORKS FOR ANY TOLERANCE WE GET

$$\boxed{\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0}$$

(THIS IS HARDER THAN ANYTHING YOU'LL NEED TO DO)

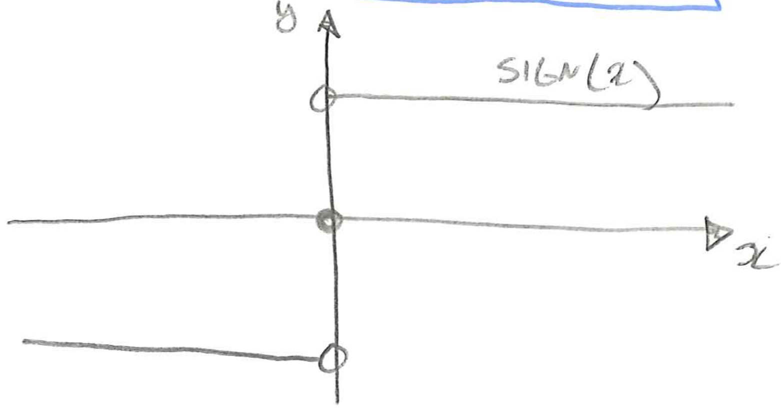
LUCKILY FOR YOU, IN THIS COURSE ONLY NEED TO PROVIDE INTUITIVE JUSTIFICATION WHEN COMPUTE LIMITS.

IX

BACK TO REASONABLE FUNCTIONS:

$$\boxed{\mathrm{SIGN}(x) = \begin{cases} 1 & \text{IF } x > 0 \\ 0 & \text{IF } x = 0 \\ -1 & \text{IF } x < 0 \end{cases}}$$

IS THIS A FUNCTION ?



WHAT IS THE FUNCTION $\begin{cases} f(x) = \mathrm{SIGN}(x) \cdot |x| \\ g(x) = \mathrm{SIGN}(x) \cdot x \end{cases}$

WHAT IS $\boxed{\lim_{x \to a} \mathrm{SIGN}(x)}$

WHEN $a \neq 0$?

$a = 0$?

X

WHY DO WE NOT TAKE INTO ACCOUNT THE VALUE OF $f(a)$ WHEN THINKING ABOUT $\lim_{x \to a} f(x)$?

GOAL: UNDERSTAND LIMITING BEHAVIOUR
SO SHOULDN'T DEPEND ON VALUE.

SOMETIMES ONLY CARE ABOUT LIMITING BEHAVIOUR ON ONE SIDE OF a

WE SAY $\lim_{x \to a^+} f(x) = L$ IF WE CAN MAKE $f(x)$ AS CLOSE AS WE LIKE TO L BY REQUIRING x TO BE SUFFICIENTLY CLOSE TO a WITH $x > a$.

SIMILAR DEFINITION FOR $\lim_{x \to a^-} f(x)$

WHAT IS $\boxed{\lim_{x \to 0^+} \operatorname{sign}(x)}$

$\boxed{\lim_{x \to 0^-} \operatorname{sign}(x)}$

$\boxed{\lim_{x \to a} f(x) \text{ EXISTS} \Updownarrow \text{WHAT?}}$

READ 2.3

$\boxed{\lim_{x \to a} f(x) = \lim_{h \to 0} f(a+h)}$