I

MIDTERM COMING UP!

FRIDAY OCTOBER 3RD 6→7 PM

GOT A CONFLICT? TALK TO ME AFTER CLASS

- PRACTISE MIDTERM + SOLUTIONS ONLINE
- MATH WIKI (PAST EXAMS + HINTS + SOLUTIONS)
- MATH 184 WORKSHOP PROBLEMS + SOLUTIONS
- STUDY GROUP PIAZZA THREAD

Q TRUE OR FALSE

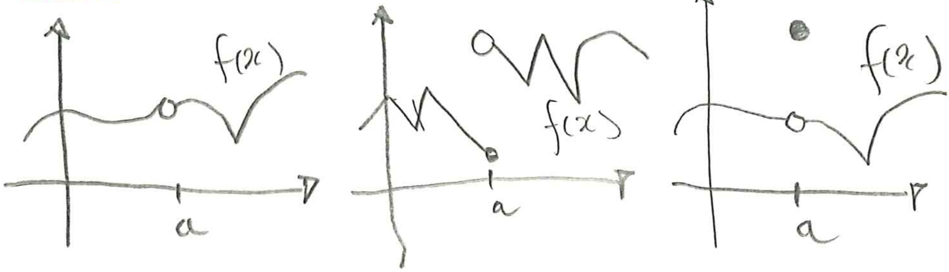
1. YOU WERE ONCE EXACTLY 3 FEET TALL.
2. AT SOME POINT SINCE YOU WERE BORN YOUR WEIGHT IN POUNDS EQUALED YOUR HEIGHT IN INCHES.
3. THE POLYNOMIAL $p(x) = x^3 - 3x + 1$ HAS A ROOT IN $[0,1]$.
4. THE POLYNOMIAL $p(x) = x^3 - 3x + 1$ HAS TWO ROOTS IN $[0,2]$.

(NO CALCULATORS)

f A FUNCTION

Q IS $\lim_{x \to a} f(x) = f(a)$?

NOT DEFINED / NOT EXIST / NOT EQUAL



THINK OF THIS ⤴ AS ABNORMAL &
GIVE NICE NAME FOR FUNCTIONS
WITH NICE PROPERTY

INTUITION: A FUNCTION f IS CONTINUOUS
IF GRAPH HAS NO BREAKS, JUMPS
OR WILD OSCILLATIONS $\left(\sin\left(\frac{1}{x}\right)\right)$

↳ LOOK AT GRAPH, CAN SEE

DEFINITION

THE FUNCTION f IS CONTINUOUS AT
THE POINT a IF $\lim_{x \to a} f(x) = f(a)$

↳ "CONTROLL x ⇒ CONTROLL $f(x)$" CAN'T ESCAPE BOSR

Q: f CONTINUOUS AT a IF THREE THINGS

HOLD: CHECKLIST.

1. f DEFINED AT a ($f(a)$ MAKES SENSE)
2. $\lim_{x \to a} f(x)$ EXISTS
3. $\lim_{x \to a} f(x) = f(a)$

IF f IS CONTINUOUS AT ALL POINTS OF ITS DOMAIN WE SAY f IS CONTINUOUS

GOAL TODAY: GET MAJOR EXPOSURE TO CONTINUITY

Q: ARE THE FOLLOWING CONTINUOUS

EASY:

- $f(x) = c$ AT 3?
- $f(x) = x$ AT 1, 0?
- $f(x) = \frac{1}{x}$ AT 1, 0?
- $f(x) = x^2$ AT 1, 0?
- $f(x) = |x|$ AT 1, 0?
- $S(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$ AT 1, 0?

WRITE LIMIT

WHAT NEEDS TO BE CHECKED?

LIMIT LAWS HAVE IMMEDIATE ANALOGUES (IV)

THM IF f & g ARE CONTINUOUS AT a

THEN ① $h(x) = f(x) + g(x)$ IS CTS AT a

② $h(x) = f(x) \cdot g(x)$ IS CTS AT a

MOREOVER, IF $g(a) \neq 0$,

THEN ③ $h(x) = \frac{1}{g(x)}$ IS CTS AT a.

Q: WHERE IS $f(x) = \frac{x^2(13x+2)}{(x^2+1)(x^2-13)}$ CONTINUOUS?

x^2, $(13x+2)$, (x^2+1) & (x^2-13) ARE CTS

(x^2+1) IS NEVER ZERO, (x^2-13) HAS TWO ROOTS

$\Rightarrow$ AT ALL POINTS EXCEPT $\pm\sqrt{13}$

LIKE WITH LIMITS STARTING FROM FACT THAT

$f(x) = x$ & $g(x) = c$ ARE CONTINUOUS

WE CAN CONCLUDE THAT

$h(x) = \frac{p(x)}{q(x)}$ ← POLYNOMIALS. IS CTS ON ITS DOMAIN

V

LIKE WITH LIMITS

CAN USE ①, ② & ③ TO GET OTHER

RULES TO SEE WHEN $c \cdot f(x)$, $(f(x))^n$ ARE CTS

OTHER FUNCTIONS TO BE AWARE OF:

① e^x

② $\ln(x)$

③ $\sin(x)$

④ $\cos(x)$

⑤ $\sqrt{x} = x^{1/2}$

⑥ $\sqrt[3]{x} = x^{1/3}$

Q: WHERE DO YOU THINK THEY ARE CONTINUOUS?

RELATIONSHIP BETWEEN FUNCTION & ITS INVERSE

IF f IS CONTINUOUS ON INTERVAL & HAS INVERSE g THEN g IS ALSO CTS (THINK OF WHAT HAPPENS TO THE GRAPH) e^x, $\ln x$

Q: WHAT DOES f CTS ON AN INTERVAL MEAN?

Q: WHAT IS AN INTERVAL?

$I = [a, b]$, $I = (a, b]$, $I = (a, b)$

f IS CONTINUOUS ON INTERVAL I

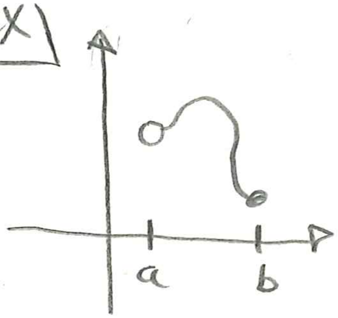
VI

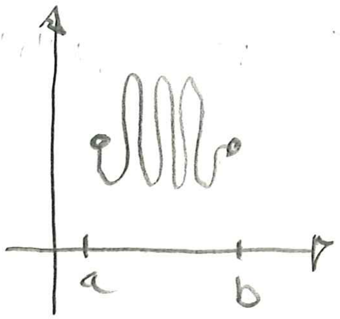
IF IT IS CONTINUOUS AT EVERY POINT INSIDE THE INTERVAL AND $\underbrace{\text{LEFT/RIGHT-CONTINUOUS}}_{\text{WHAT?}}$ AT EVERY INCLUDED END-POINT

[a, b] (a, b] (a, b)

INCLUDED NOT INCLUDED

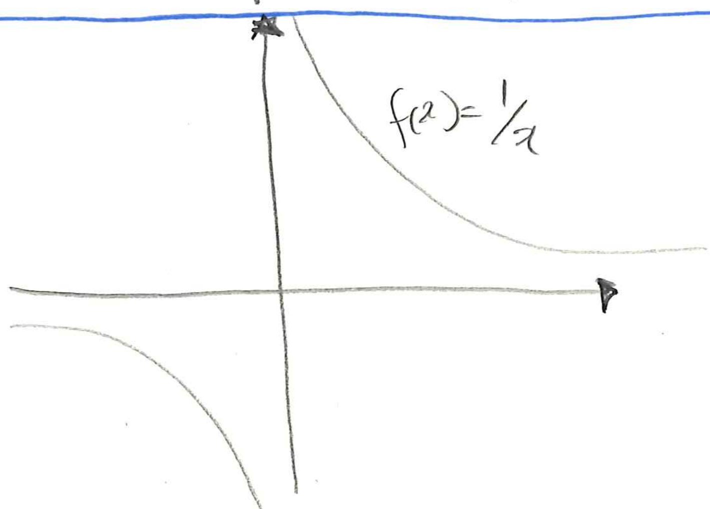
EX)





Q: IS $f(x) = \frac{1}{x}$ CTS ON

[0,1] ? [0,1) ? (0,1] ? (0,1) ?



VII

ANOTHER WAY TO COMBINE CONTINUOUS FUNCTIONS & OBTAIN OTHER CONTINUOUS FUNCTIONS $f(x)$, $g(x)$ $\Rightarrow$ $(f\circ g)(x) = f(g(x))$?

THM: IF $f(x)$ & $g(x)$ ARE CTS AT a THEN $(f\circ g)(x) := f(g(x))$ IS CTS AT a.

"x CONTROLS $g(x) = y$ WELL
& y CONTROLS $f(y)$ WELL
$\Rightarrow$ x CONTROLS $f(g(x))$"

$$x \to \boxed{g} \to g(x) = y \to \boxed{f} \to f(g(x)) = f(y)$$

EX $(x^3+1)^2 = (x^3+1)(x^3+1)$

CTS

SO PRODUCT IS CTS

ANOTHER WAY TO SEE IT

$$x \to \boxed{g} \to g(x) = x^3+1 = y \to \boxed{f} \to f(g(x)) = (x^3+1)^2$$

$\begin{cases} f(y) = y^2 \\ g(x) = x^3+1 \end{cases}$ BOTH CTS

SO

$f\circ g$ CTS

THIS WILL BECOME VERY IMPORTANT: MASTER IT.

VIII

EX $|(x^3+1)| = h(x)$ IS CTS

$x \to \boxed{g}$

$\to g(x) = (x^3+1) \to \boxed{f}$

$\to f(g(x)) = |x^3+1|$

$$\begin{cases} f(y) = |y| \\ g(x) = x^3+1 = y \end{cases}$$

BOTH CTS SO $f \circ g$ CTS

EX $\sqrt{x^3+1} = h(x)$

$x \to \boxed{g}$

$\to g(x) = x^3+1 \to \boxed{f}$

$\to f(g(x)) = \sqrt{x^3+1}$

$f(y) = \sqrt{y}$

CTS FOR $\boxed{y \geq 0}$

$g(x) = x^3+1 = y$

CTS EVERYWHERE.

NEED $g(x) = x^3+1 = y \geq 0$

NEED $x^3 \geq -1$ ie $x \geq \sqrt[3]{-1} = -1$

So $h(x) = \sqrt{x^3+1}$ IS CTS ON DOMAIN $x \geq -1$

EX] $h(x) = \sqrt{|x^3+1|}$ CTS EVERYWHERE

$x \to \boxed{g}$

$\to g(x) = |x^3+1| \to \boxed{f}$

$\to f(g(x)) = \sqrt{|x^3+1|}$

$f(y) = \sqrt{y}$

CTS FOR $y \geq 0$

$g(x) = |x^3+1| = y$

ALWAYS ≥ 0

EX] e^{x^3+1}, $\sin(2^{\ln(x)})$, $\cos\left(\frac{1}{x}\right)$

WHERE ARE THEY CONTINUOUS?

SIGN(x^3+1) CTS? WHERE?

WHAT'S THE POINT OF ALL THIS?

LET f BE A CONTINUOUS FUNCTION ON THE CLOSED INTERVAL $[a, b]$ WITH $f(a) < 0$ & $f(b) > 0$.

CAN YOU ALWAYS FIND A POINT $a < c < b$ WITH $f(c) = 0$?

YES / NO / WHY !!! USE EXAMPLES & PICTURES TO GUIDE YOUR THINKING

→ WRITE 2, 3 SENTENCES AS ANSWER.