I

NO OFFICE HOUR THURSDAY

→ FRIDAY 1→2 PM INSTEAD

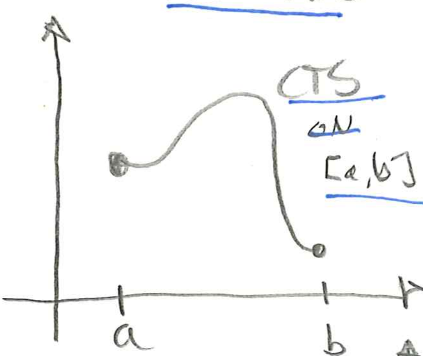
OR TODAY 3:30 → 4:30 PM

HAND IN NOW

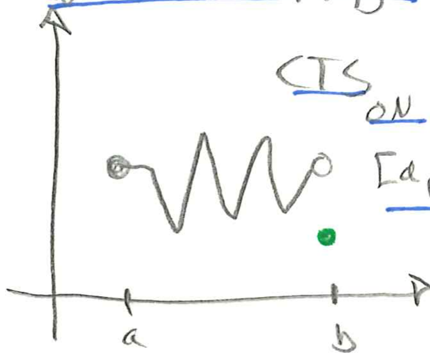
LAST TIME

- f CTS AT c IF $\lim_{x \to c} f(x) = f(c)$
- WILL SAY f CTS ON INTERVAL I IF IT IS CTS AT EVERY INTERIOR POINT OF I & CTS FROM THE LEFT/RIGHT AT INCLUDED END POINTS

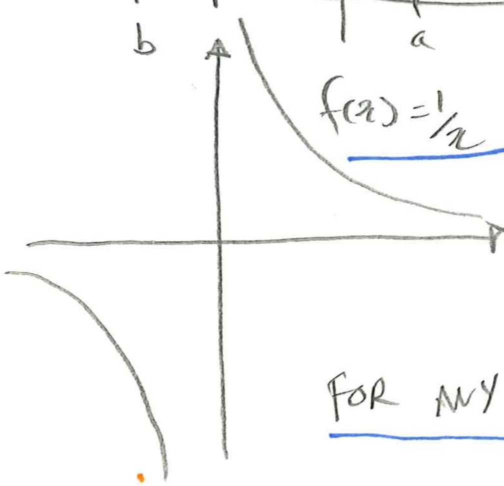
E.G. $[a, b]$ INCLUDED → $[a, b)$ NOT INCLUDED



CTS ON $[a,b]$



CTS ON $[a,b)$



$f(x) = 1/x$ CTS ON $(0, b]$ & $[a, 0)$

FOR ANY $b > 0$ & $a < 0$

BUILT UP A "ZOO" OF CTS FUNCTIONS $\boxed{\text{I}}$

$f(x) =$ POLYNOMIAL, e^x, $\ln(x)$, $\sqrt{x}$, $\sqrt[3]{x}$, $|x|$

Q: INTERVALS OF CONTINUITY?

FOUND WAYS TO COMBINE CTS FUNCTIONS TO MAKE NEW ONES: f & g CTS

$\Rightarrow f+g$, $f \cdot g$, $f \circ g$ & $\frac{f}{g}$ (WHEN $g \neq 0$)

THE BIG $\boxed{\text{QUESTION}}$: WHY DO WE CARE ABOUT CTS FUNCTIONS !?

$\boxed{\text{ANSWER}}$ OUR FIRST BIG THEOREM

THE INTERMEDIATE VALUE THEOREM

IF f IS CONTINUOUS ON $[a, b]$

AND $f(a) < 0 < f(b)$

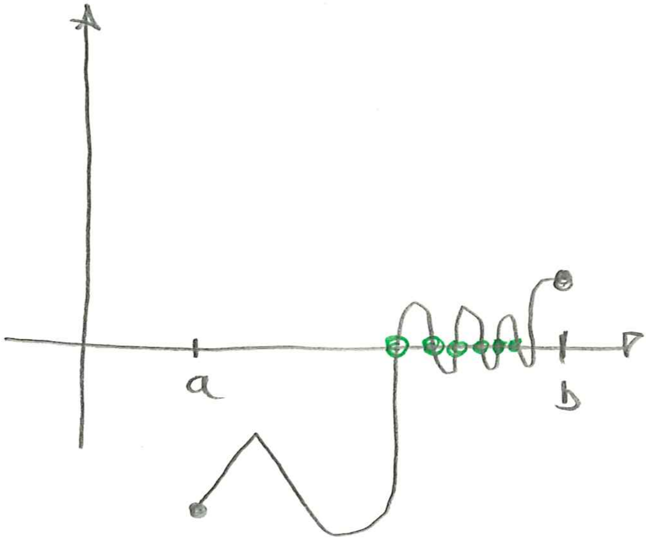
THEN THERE IS SOME c $(a < c < b)$

SUCH THAT $f(c) = 0$

GEOMETRIC INTERPRETATION:

GRAPH OF A CTS FUNCTION ON $[a,b]$ STARTING BELOW HORIZONTAL AXIS & ENDING ABOVE IT MUST CROSS THE AXIS AT LEAST ONCE

NOTICE: DESCRIBES GLOBAL BEHAVIOUR OF A FUNCTION RATHER THAN JUST AT POINT



IN ASSIGNMENT TRIED TO COME UP WITH AN EXPLANATION

NOTICE: NOTHING SPECIAL ABOUT 0

IF f IS CTS ON $[a,b]$ & $f(a) < \alpha < f(b)$ THEN THERE IS SOME $a < c < b$ WITH $f(c) = \alpha$ (OR ANY NUMBER)

WHY: LET $g(x) = f(x) - \alpha$

THEN g IS CTS & $g(a) < 0 < g(b)$

BY IVT CAN FIND $a < c < b$ WITH $g(c) = 0$

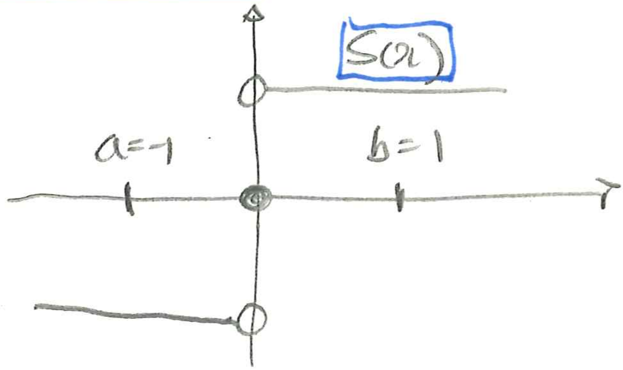
BUT $g(c) = f(c) - \alpha = 0$

$\Rightarrow f(c) = \alpha$

NOTICE: IF f IS DISCONTINUOUS AT EVEN A SINGLE POINT IN $[a,b]$ THE RESULT IS FALSE

IV

EX



WHY IS THIS A BIG DEAL? IVT

LET'S GO BACK TO OUR Q'S

① WERE ONCE EXACTLY 3 FEET TALL

$h(t)$ = HEIGHT (TIME) ↳ WHAT YOU THINK I WILL DO

ON $[0, 20]$ YEARS

$h(0) < 3$ & $h(20) > 3$

$\Rightarrow$ THERE IS $0 < c < 20$

$h(c) = 3$ BY IVT

② WEIGHT (TIME) = HEIGHT (TIME) Ⅴ

POUNDS INCHES

WANT $\boxed{f(t) = w(t) - h(t) = 0}$

$w(0) < 14$ POUNDS AT BIRTH 5.5 → 10

$h(0) > 14$ INCHES AT BIRTH 18 → 22

$\Rightarrow f(0) = w(0) - h(0) < 0$

$w(20) > 90$ POUNDS

$h(20) < 90$ INCHES (7.5 FEET, 2.2 M)

$\Rightarrow f(20) = w(20) - h(20) > 0$

$\Rightarrow$ AT SOME TIME c

$f(c) = 0 = w(c) - h(c)$

$\Rightarrow w(c) = h(c)$

CHALLENGE

AT ANY POINT IN TIME, THERE ARE TWO DIAMETRICALLY OPPOSITE POINTS ON THE EQUATOR WITH PRECISELY THE SAME TEMP

LET $p(x) = x^3 - 3x + 1$ (VI)

① HAS A ROOT IN $[0,1]$ ANY IDEAS?

$p(0) = 0^3 + 3 \cdot 0 + 1 = 1 > 0$

$p(1) = 1^3 - 3 \cdot 1 + 1 = 1 - 3 + 1 = -1 < 0$

$\Rightarrow$ FOR SOME $0 < c < 1$ BY IVT
WE HAVE $p(c) = 0$

(GOOD LUCK TRYING TO FIND EXACT VALUE!)

② HAS TWO ROOTS IN $[0,2]$

$p(2) = 2^3 - 3 \cdot 2 + 1 = 8 - 6 + 1 = 1 > 0$

$p(1) = -1 < 0$

$\Rightarrow$ FOR SOME $1 < d < 2$ WE HAVE $p(d) = 0$ BY IVT

$\Rightarrow$ YES $p(x)$ HAS (AT LEAST) TWO ROOTS IN $[0,2]$ & CHALLENGE

EXAMPLE 9 SECTION 2.6 (NEED TO KNOW)

THIS IS WHERE PRECALCULUS ENDS & THE POWERFUL IDEAS OF CALCULUS BEGIN

VII

GENERAL FUNCTIONS ARE TOO $\boxed{\text{WILD}}$ TO UNDERSTAND COMPLETELY

$$f(x) = \begin{cases} e^{x} & \text{RATIONAL} \\ \sin\left(\frac{1}{x}\right) & \text{IRRATIONAL} \end{cases}$$

CONTINUOUS FUNCTIONS ARE SLIGHTLY NICER

$g(x) = |x|$

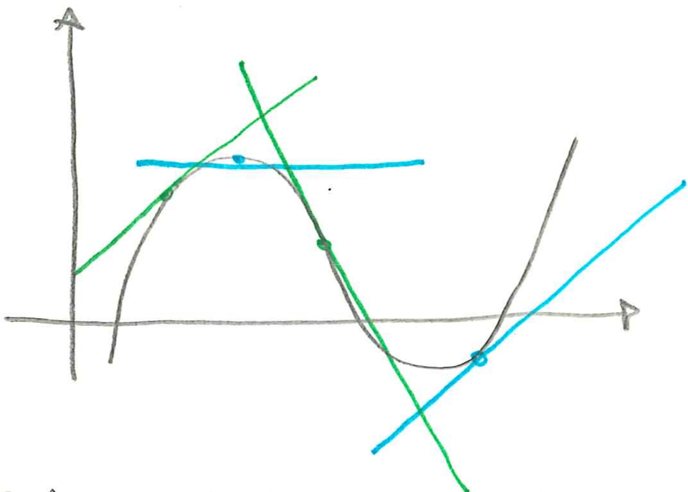
BUT STILL HARD $h(x) = \sin\left(\frac{1}{x}\right)$

WE WANT SOMETHING EVEN BETTER

GOAL: USE EASY FUNCTIONS (LINEAR, POLYNOMIAL) TO UNDERSTAND HARDER ONES.

→ WANT A NICE TYPE OF FUNCTIONS FOR WHICH IT IS POSSIBLE

FIRST STEP: USE TANGENT LINES (VIII)



WHAT DO WE EVEN MEAN BY THIS?

Q AT (0,0) THE GRAPH OF $f(x)=|x|$

HAS (A) A TANGENT LINE $y(x)=0$

(B) INFINITELY MANY TANGENTS

(C) NO TANGENT LINE

(D) TWO TANGENT LINES $y(x)=x$, $y(x)=-x$

?

Q THE TANGENT LINE TO THE GRAPH OF $f(x)=x$ AT $x=0$

(A) IS $y=0$

(B) IS $y(x)=x$

(C) DNE

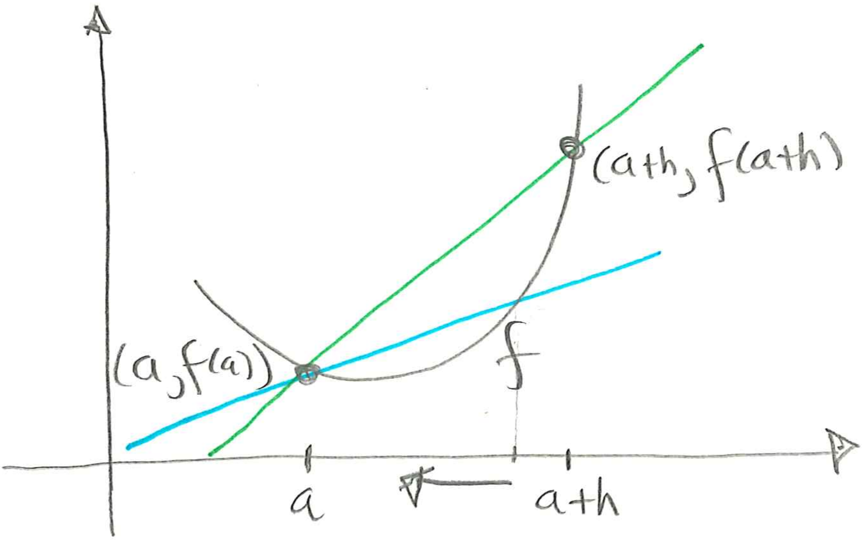
(D) IS NOT UNIQUE, ∞-LY MANY TANGENTS

?

IX

NEED TO AVOID AMBIGUITY & MAKE SURE WE KNOW WHAT WE ARE TALKING ABOUT!

START WITH SECANTS & USE LIMITS



DON'T KNOW HOW TO TALK ABOUT A "LIMIT" OF LINES, BUT WE CAN MAKE SENSE OF A LIMIT OF SLOPES

SLOPE: $\dfrac{\Delta y}{\Delta x} = \dfrac{f(a+h) - f(a)}{(a+h) - a} = \dfrac{f(a+h) - f(a)}{h}$

TAKE $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

READ 3.1 & 3.2

WHAT DO WE GET, HOW TO USE IT?

CHALLENGE PROBLEM.