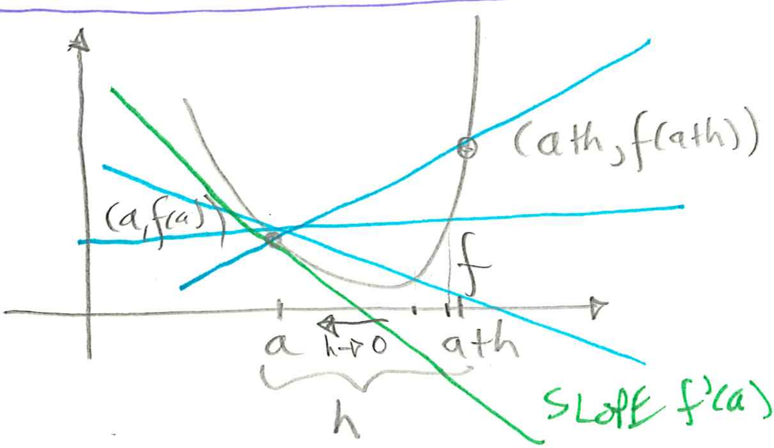
HAVE YOU LOOKED AT THE LEARNING GOALS FOR WEEK 2 ? (TO KNOW WHAT YOU MIGHT BE ASKED ABOUT ON THE MIDTERM FRIDAY OCTOBER 3RD @ 6 PM)

I

OFFICE HOURS TODAY 1→2 PM IN LSK 300C / MLC

LAST TIME: GOAL WAS TO USE LINES TO UNDERSTAND BEHAVIOUR (SHAPE OF GRAPH) OF FUNCTIONS

↳ TANGENT LINES



DON'T KNOW HOW TO TAKE A "LIMIT" OF LINES, BUT WE DO KNOW HOW TO TAKE A LIMIT OF SLOPES

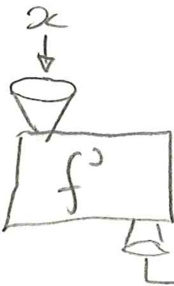
$$\frac{\Delta y}{\Delta x} = \frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$$

SLOPE OF TANGENT LINE "SHOULD" BE

$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

IF IT EXISTS, WE CALL IT THE DERIVATIVE OF f AT a, IN SYMBOLS: $f'(a)$ AND SAY f IS DIFFERENTIABLE AT a. IN THIS CASE, WE DEFINE THE TANGENT LINE TO THE GRAPH OF f AT $(a, f(a))$ TO BE THE LINE THROUGH $(a, f(a))$ WITH SLOPE $f'(a)$.

NOTICE: IF f IS DIFFERENTIABLE ON AN INTERVAL CAN THINK OF f' AS A FUNCTION



CALL THIS FUNCTION THE DERIVATIVE OF f

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

THE DERIVATIVE (IF IT EXISTS) ENCODES THE SLOPES OF ALL THE TANGENT LINES TO THE GRAPH OF A FUNCTION ! ORGANIZED NICELY INTO A FUNCTION ∴ IT SHOULD BE TELLING US SOMETHING ABOUT THE SHAPE OF THIS GRAPH. HOLD THAT THOUGHT, LET'S SEE IF WE CAN ACTUALLY FIGURE OUT WHAT IT IS FOR OUR FAVOURITE FUNCTIONS

- $\boxed{f(x) = c}$

$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h \to 0} \frac{c-c}{h} = 0$$

$$\Rightarrow f'(a) = 0$$

Q DOES THIS MAKE SENSE?

- $\boxed{f(x) = x}$

$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h \to 0} \frac{(a+h)-(a)}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$$

Q: DOES THIS MAKE SENSE?

IV

- $\boxed{f(x) = x^3}$

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(a+h)^3 - a^3}{h}$$

$$= \lim_{h\to 0} \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h}$$

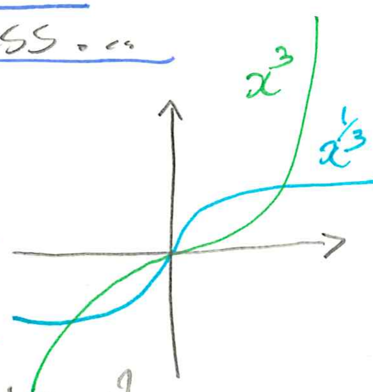
$$= \lim_{h\to 0} \frac{3a^2h + 3ah^2 + h^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3a^2 + 3ah + h^2)}{\cancel{h}}$$

$$= \lim_{h\to 0} 3a^2 + 3ah + h^2 = 3a^2$$

HARDER TO GUESS...

- $\boxed{f(x) = x^{1/3} = \sqrt[3]{x}}$



Q: IS THIS FUNCTION CTS?

Q: DOES IT HAVE A TANGENT AT 0?

Q: IS IT DIFFERENTIABLE AT 0?

$$\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\sqrt[3]{h} - \sqrt[3]{0}}{h}$$

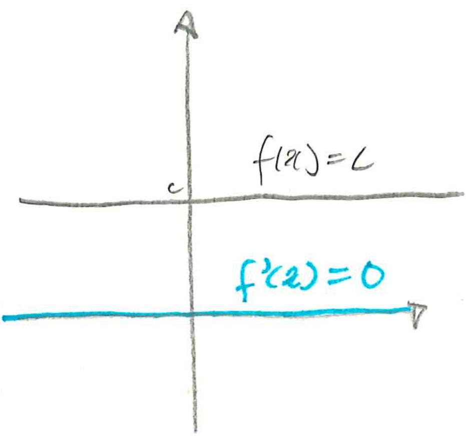
$$= \lim_{h\to 0} \frac{h^{1/3}}{h} = \lim_{h\to} h^{1/3 - 1} = \lim_{h\to 0} \frac{1}{h^{2/3}} \quad \boxed{\text{DNE}}$$

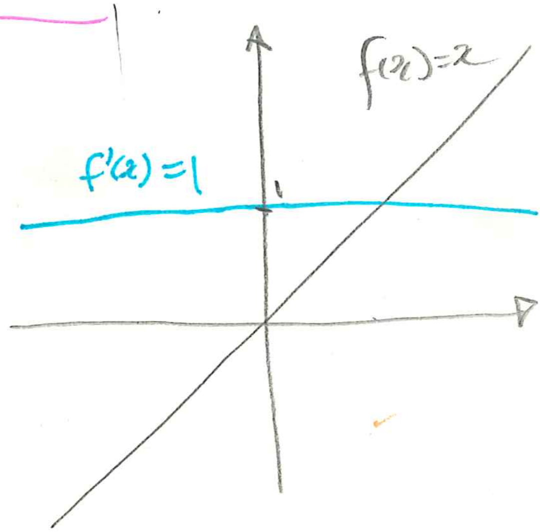
WE MADE OUR DEFINITION, NOW WE HAVE TO LIVE WITH IT

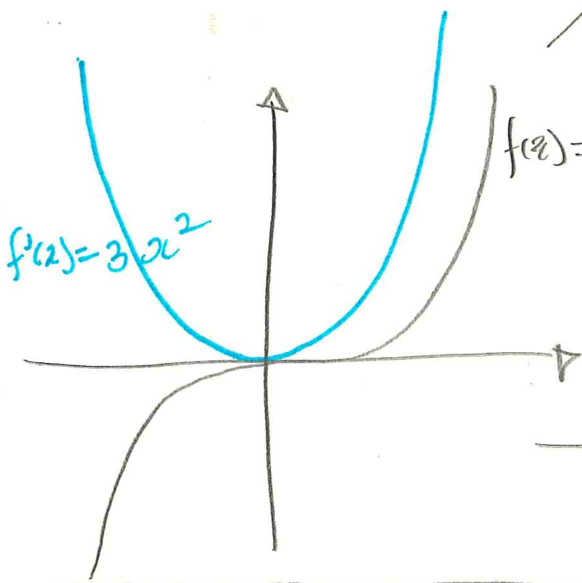
Q: WHY DOES IT MAKE SENSE TO RULE OUT VERTICAL "TANGENT" LINES? (V)

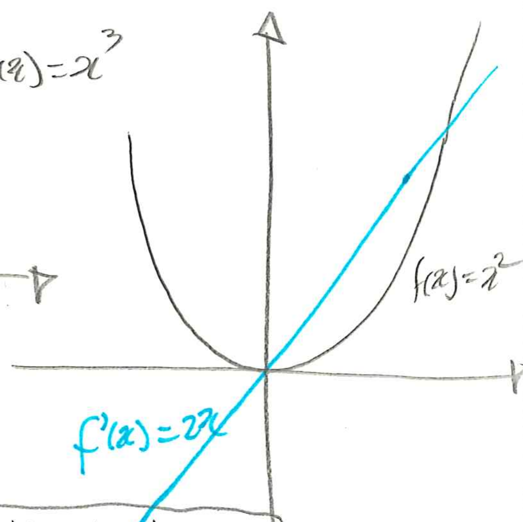
↳ WANT THE DERIVATIVE TO BE A FUNCTION WHICH ENCODES THE SHAPE OF THE GRAPH OF f. :

WHAT DO I MEAN?

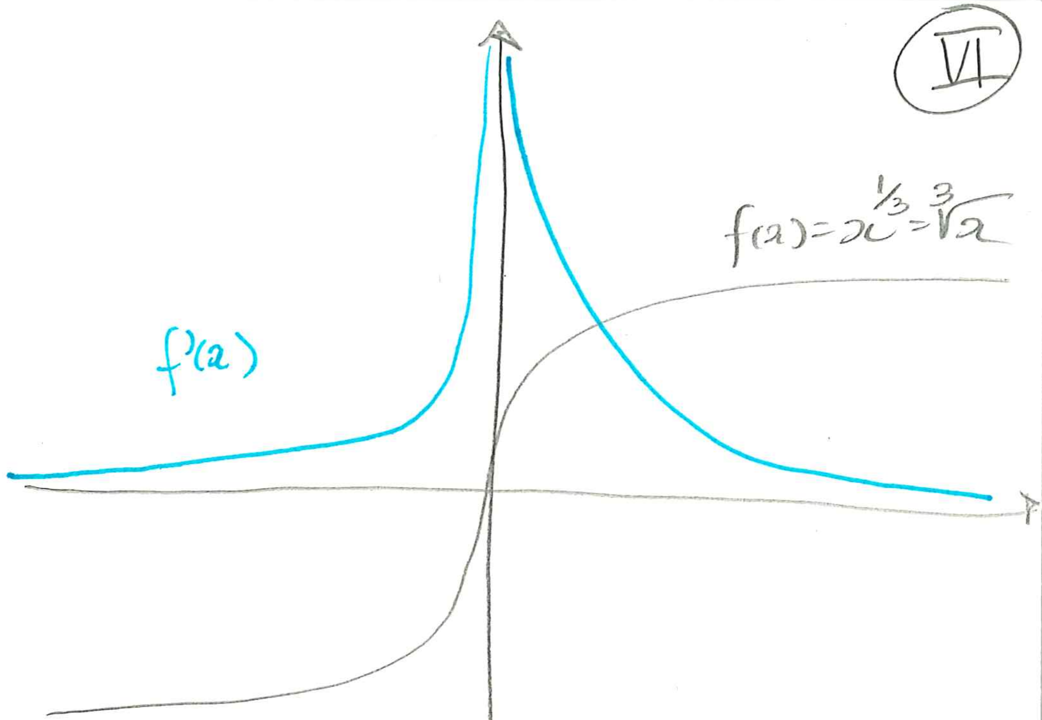








TAKES PRACTISE VISUALIZING



SKIP?

- $\boxed{f(x) = |x|}$ $\boxed{Q}$: CTS?

$\boxed{Q}$: DIFFERENTIABLE AT 1?

$$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{|1+h| - |1|}{h}$$

$$= \lim_{h \to 0} \frac{1+h-1}{h} = \lim_{h \to} \frac{\cancel{h}}{\cancel{h}} = 1$$ YES

Q: DIFFERENTIABLE AT 0?

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{|h|}{h}$$

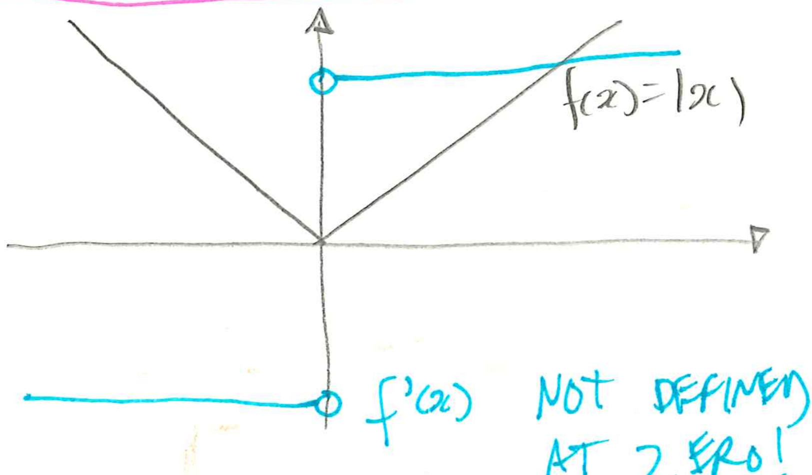
NEED TO DO LEFT/RIGHT LIMITS

$$\lim_{h \to 0^+} \frac{|h|}{h} = \lim_{h \to 0^+} \frac{h}{h} = 1$$

$$\lim_{h \to 0^-} \frac{|h|}{h} = \lim_{h \to 0^-} \frac{-h}{h} = -1$$

SO $\boxed{\text{DNE}}$

WHAT ARE WE SEEING?



A DERIVATIVE IS OFTEN A NASTIER FUNCTION THAN WHAT YOU START WITH...

$\boxed{\text{Q}}$: IS THERE A RELATIONSHIP BETWEEN CONTINUOUS FUNCTIONS & DIFFERENTIABLE FUNCTION

$\boxed{\text{Q}}$ ARE ALL CTS FUNCTIONS DIFFERENTIABLE?

Q: IF f IS DIFFERENTIABLE AT a, DOES IT IMPLY THAT f IS CTS AT a?

VIII

THM: DIFFERENTIABLE ⇒ CONTINUOUS

IF f IS DIFF AT a, THEN IT IS ALSO CTS AT a

↳ READ PROOF IN TEXT HOMEWORK

LOGIC WHAT DO WE MEAN WHEN WE SAY

"A IMPLIES B" "A ⇒ B"

IF A IS TRUE THEN B MUST BE TRUE

I STAND NAKED IN THE SUN FOR HOURS ⇒ I WILL BE SUNBURNT True

IT IS RAINING ⇒ I AM WET False

↳ MAYBE YOU STAYED INSIDE

THE INTEGER n IS EVEN ⇒ $(n+1)$ IS AN ODD INTEGER TRUE

THE INTEGER n IS EITHER 1 OR 2 ⇒ $(n+1)$ IS ODD FALSE

NOT ALWAYS