MIDTERM FRIDAY OCT 3RD @ 6PM

I

↳ CONFLICT (...)? TALK TO ME AFTER

↳ DO PRACTISE MIDTERM!

OFFICE HOURS { THURSDAY 2:00 → 3:00 PM
FRIDAY 1:00 → 2:00 PM

LAST TIME

FOUND RULES TO DIFFERENTIATE SUMS, PRODUCTS & QUOTIENTS OF DIFFERENTIABLE FUNCTIONS

SUM RULE

IF $h(x) = f(x) + g(x)$ AND BOTH DIFF @ x

THEN $h'(x) = f'(x) + g'(x)$

PRODUCT RULE

IF $h(x) = p(x) \cdot q(x)$ AND BOTH DIFF @ x

THEN $h'(x) = p'(x)q(x) + p(x)q'(x)$

QUOTIENT RULE

IF $h(x) = \dfrac{u(x)}{v(x)}$ AND BOTH DIFF @ x, $v(x) \neq 0$

THEN $h'(x) = \dfrac{u'(x)v(x) - u(x)v'(x)}{(v(x))^2}$

USEFUL SPECIAL CASES

II

CONSTANT MULTIPLE RULE

IF $h(x) = c \cdot f(x)$ AND f DIFF AT x

THEN $h'(x) = c \cdot f'(x)$

POWER RULE

IF n IS A NON-NEGATIVE INTEGER

AND $h(x) = x^n$ THEN $h'(x) = n x^{n-1}$

TRUE OR FALSE

IF $h(x) = p(x) \cdot q(x)$ WHERE $p(x)$ IS DIFF

BUT $q(x)$ IS NOT DIFF AT a

THEN $h(x)$ IS NOT DIFF AT a

$\boxed{h(x) = x \cdot |x| \quad \text{CAUTION}}$

TRUE OR FALSE

THE POWER RULE ONLY HOLDS FOR

NON-NEGATIVE INTEGERS

(JUSTIFY YOUR ANSWERS!)

$h(x) = x^{-3} = \frac{1}{x^3}$ ⇝ QUOTIENT RULE

$u(x) = 1 \quad u'(x) = 0$

$v(x) = x^3 \quad v'(x) = 3x^2$

$$\frac{-3x^2}{(x^3)^2} = \frac{-3}{x^4} = -3x^{-4}$$

EXTENDED POWER RULE

FOR ANY INTEGER n

IF $h(x) = x^n$ THEN $h'(x) = n x^{n-1}$

... AND THAT'S IT FOR THE RULES

KNOWN DIFFERENTIABLE FUNCTIONS + RULES

=

UNIVERSE OF FUNCTIONS WE CAN HANDLE

Q HOW CAN WE EXPAND THIS UNIVERSE?

$h(x)$	$h'(x)$
$1+x$	1
$1+x+\frac{1}{2}\cdot x^2$	$1+x$
$1+x+\frac{1}{2}x^2+\frac{1}{6}x^3$	$1+x+\frac{1}{2}x^2$
$1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{12}$	$1+x+\frac{x^2}{2}+\frac{x^3}{6}$
$1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{12}+\frac{x^5}{60}$	$1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{12}$

$\boxed{Q}$ LET $f(x) = e^x$ IV

- DO YOU THINK IT IS DIFFERENTIABLE?
- WHAT DO YOU THINK $f'(x)$ SHOULD BE?

INTRODUCED e AS THE MAGICAL NUMBER FOR WHICH $e^x = 1 + x + \frac{x^2}{2} + \cdots + \frac{x^k}{k!} + \cdots$

& CONVINCED OURSELVES THIS WAS PLAUSIBLE BY PLOTTING POLYNOMIALS

(COULDN'T MAKE IT PRECISE & STILL CAN'T)

TODAY WE REDISCOVER e IN A NEW WAY:

$\boxed{Q}$
- IF $f(x) = b^x$ $(0 < b \neq 1)$?
- CAN YOU USE RULES TO FIND $f'(x)$?
- HOW CAN YOU FIND $f'(x)$ IF IT EXISTS

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{b^{x+h} - b^x}{h}$$

OK NOW?

WHAT

IV

$$\boxed{f(x) = b^x}$$

$$\lim_{h \to 0} \frac{b^{(x+h)} - b^x}{h}$$

$$= \lim_{h \to 0} \frac{b^x b^h - b^x}{h}$$

$$= \lim_{h \to 0} \frac{b^x (b^h - 1)}{h}$$

$$= b^x \underbrace{\lim_{h \to 0} \frac{b^h - 1}{h}}$$

$\boxed{Q}$ WHAT IS THIS?

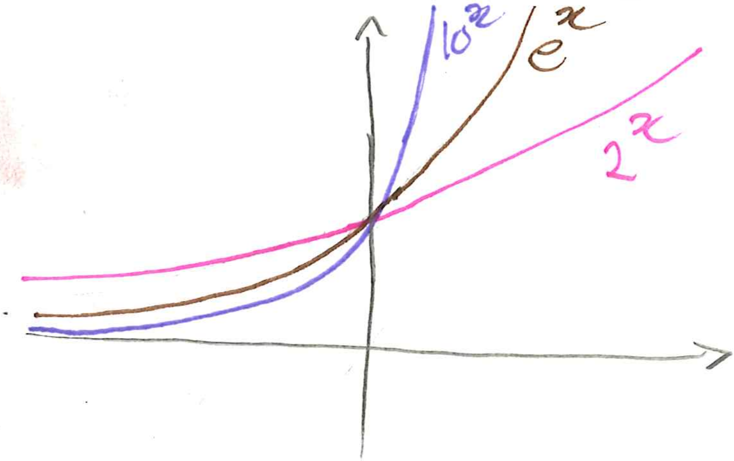
$$\boxed{\lim_{h \to 0} \frac{b^{0+h} - b^0}{h} = f'(0)}$$

CAN SHOW $\lim_{h \to 0} \frac{b^h - 1}{h}$ EXISTS $0 < b$

$$\Rightarrow \boxed{f'(x) = b^x \cdot f'(0) \text{ WHEN } f(x) = b^x}$$

IF $f'(0) = 1$ SOMETHING MAGICAL HAPPENS, $f'(x) = b^x$

CAN WE FIND A BASE b WHICH MAKES THIS HAPPEN?



THERE IS A UNIQUE NUMBER FOR WHICH THIS HAPPENS & WE GIVE IT THE NAME "e".

THE DEFINING PROPERTY IS THAT

$$\boxed{\lim_{h\to 0}\frac{e-1}{h}=1}$$

e IS THE NUMBER WHICH MAKES THIS WORK

IF $h(x)=e^x$

THEN $h'(x)=e^x$

Q: HOW MUCH HAVE WE GAINED?

$\boxed{Q}$: CAN WE DIFFERENTIATE $f(x)=e^{2x}$ VII

$f(x) = e^{2x} = e^{x+x} = e^{x}e^{x}$ $\boxed{\text{PRODUCT RULE}}$

$f'(x) = e^{x}e^{x} + e^{x}e^{x} = 2e^{2x}$

↳ GENERALIZES $\boxed{f(x)=e^{kx} \Rightarrow f'(x)=ke^{kx}}$

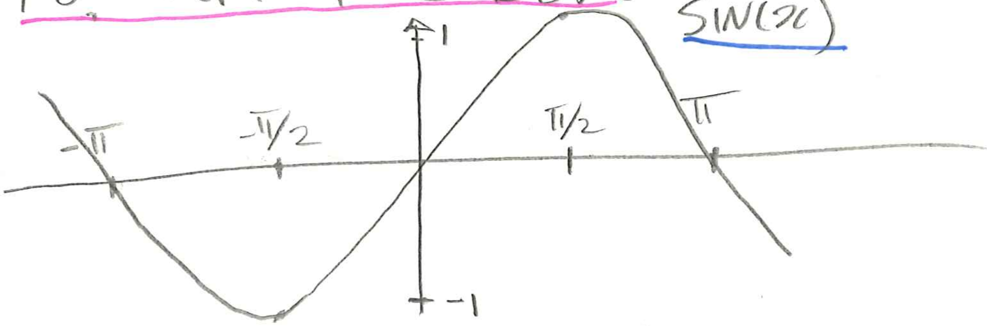
(HOW CAN YOU DO IT FOR REAL k?)

EX| $f(x) = xe^{x}$ IS IT DIFF?

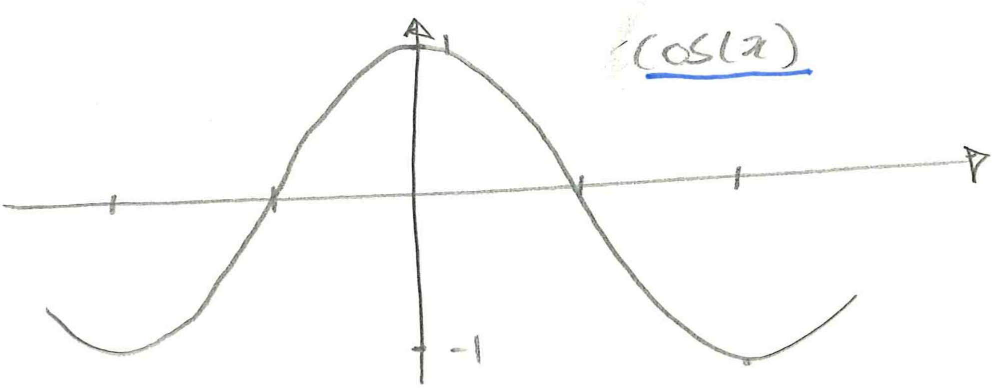
$f'(x) = 1\cdot e^{x} + xe^{x} = (1+x)e^{x}$

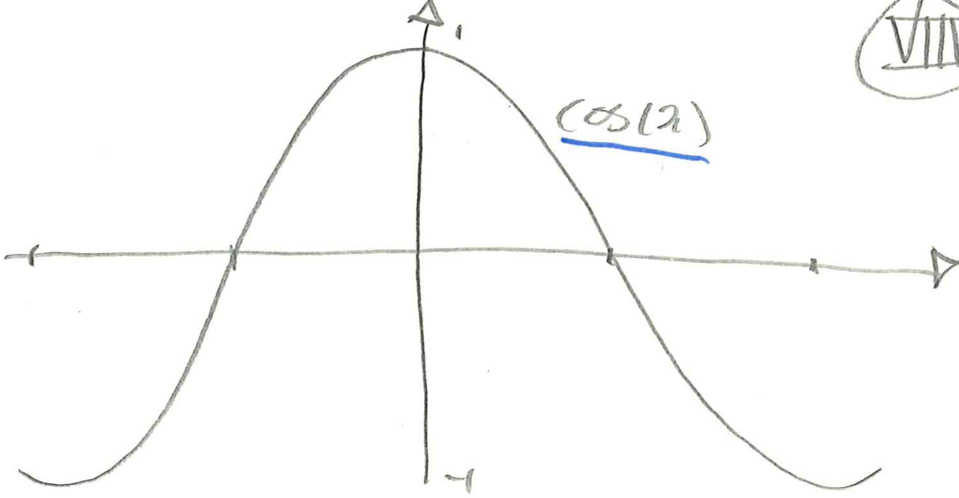
LET'S ADD TRIGONOMETRIC FUNCTIONS TO OUR TOOL BOX

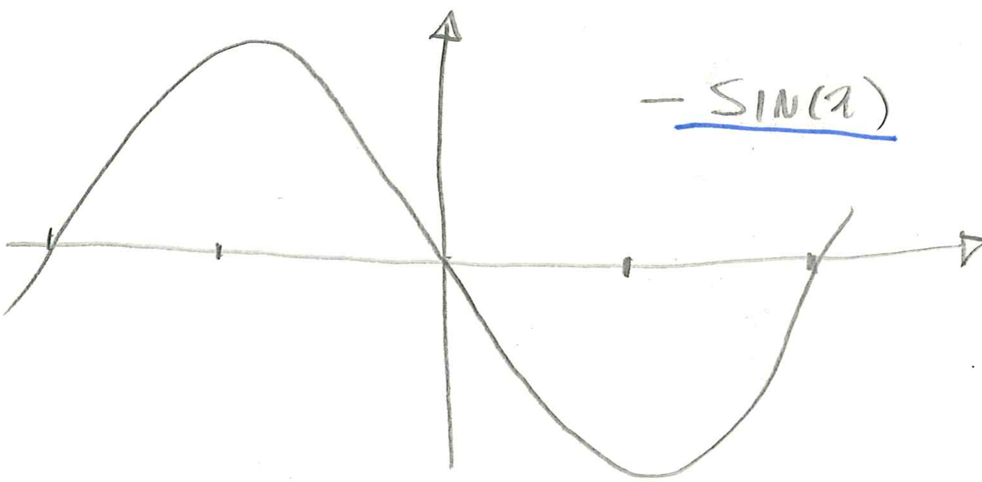
SIN(x)



COS(x)







THM: $f(x) = \sin(x) \qquad g(x) = \cos(x)$

$$f'(x) = \cos(x) \qquad g'(x) = -\sin(x)$$

$\boxed{Q}$: CAN WE HANDLE $f(x) = \sin(2x)$?

NO

$$\cancel{\sin(2x) = \sin(x)\sin(x)}$$

$$\cancel{\sin(2x) = \sin(x) + \sin(x)}$$

Q: CAN WE HANDLE $h(x) = \tan(x)$? IX

QUOTIENT RULE

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \Rightarrow h'(x) = \frac{\cos(x)\cos(x) + \sin(x)\sin(x)}{\cos(x)\cos(x)}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$

CAUTION

$$\boxed{\sin^2(x) = (\sin(x))^2}$$

STUPID NOTATION

REAL MEANING

FRIDAY'S CLASS

- DETAILS ON MIDTERM CONTENT
- HOW TO SOLVE PROBLEMS
- QUIZ (PARTICIPATION MARKS)

(DON'T STUDY FOR IT... THE LECTURE WILL BE YOUR STUDY)