

IMPORTANT SURVEY ON PIAZZA

STUDENT GUIDE

HTTP://BLOGS.UBC.CA/MATH STUDENT GUIDE 104

Q  $\frac{d}{dx}(e^7)$  (A) =  $7e^6$   
(B) =  $e^7$   
(C) = 0

Q IF  $f(1)=1$  &  $f'(1)=3$  THEN

$\frac{d}{dx}\left(\frac{f(x)}{x^2}\right)$  AT  $x=1$  EQUALS

- (A) 1
- (B) 3/2
- (C) -1

T/F BY CONSTANT RULE  $\frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x))$

BY PRODUCT RULE

$\frac{d}{dx}(cf(x)) = \left(\frac{d}{dx}(c)\right) \cdot f(x) + c \cdot \frac{d}{dx}(f(x))$

& BOTH RULES GIVE SAME RESULT

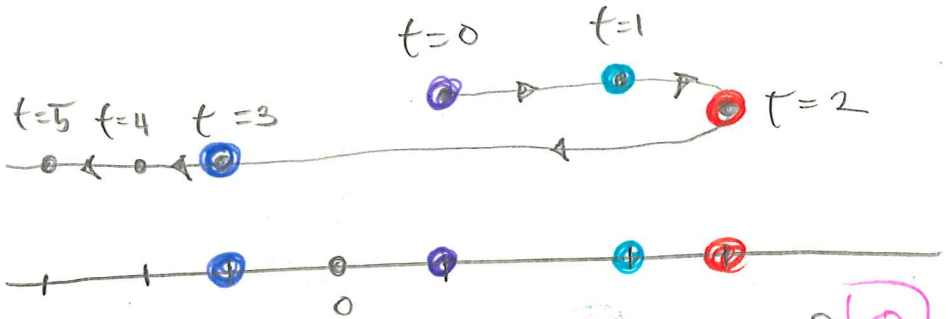
(II)

CONSIDER PARTICLE MOVING ON  
A STRAIGHT LINE WITH A CHOSEN

ORIGIN O

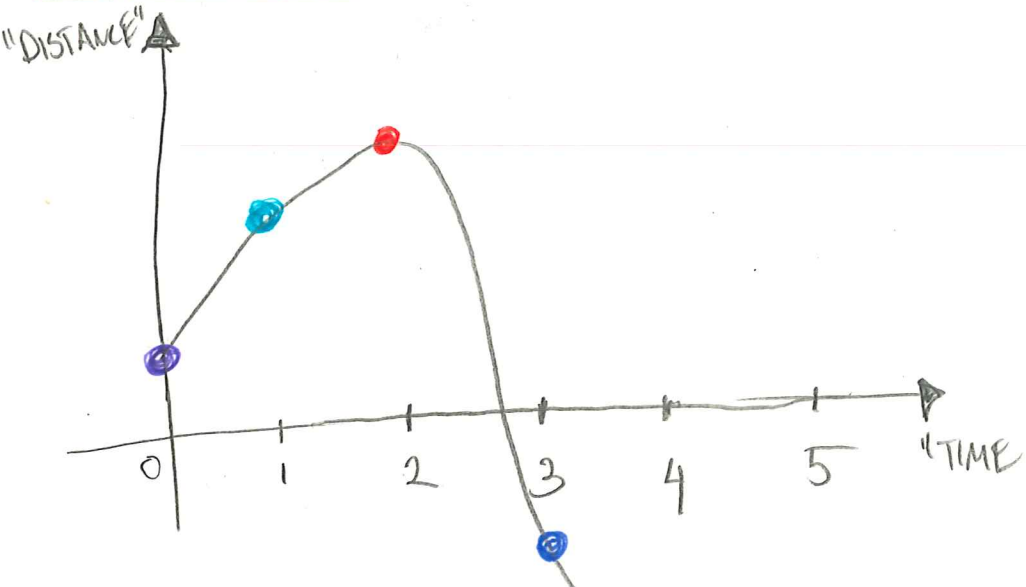
POSITIVE DIRECTION

LET  $s(t)$  DENOTE DISTANCE OF PARTICLE  
FROM O AT TIME  $t$



GRAPH  $s(t)$

HOW DOES IT LOOK? Q



WHAT DOES

Q

III

$\frac{s(a+h) - s(a)}{h}$  REPRESENT FOR

FIXED  $a$  &  $h > 0$  ?

AVERAGE VELOCITY DURING INTERVAL  
 $[a, a+h]$  (DEPENDS ON  $h$ )

WHAT DOES

$\lim_{h \rightarrow 0} \frac{s(a+h) - s(a)}{h}$  REPRESENT

Q

FOR FIXED  $a$  ?

INSTANTANEOUS VELOCITY (INDEP OF  $h$ )

↳ THEORETICAL CONCEPT LIMIT OF  
AVERAGE VELOCITIES

↳ RATE OF CHANGE OF POSITION

Q) CAN YOU THINK OF OTHER

Q

EXAMPLES WHERE DERIVATIVES WOULD  
COME INTO PLAY ?

# RECALL BUSINESS PROBLEM

(IV)

SETUP:

$C(x)$  = PRODUCTION COST FOR  $x$  ITEMS

THINGS YOU MIGHT WORRY ABOUT:

↳ AVERAGE COST  $\bar{C}(x) = \frac{C(x)}{x}$

HOW DOES THIS QUANTITY CHANGE WITH  $x$ ?

↳ AVERAGE COST OF PRODUCING

$\Delta x$  MORE UNITS GIVEN THAT

HAVE ALREADY PRODUCED  $x$  UNITS

$$\frac{\Delta C}{\Delta x} = \frac{C(x + \Delta x) - C(x)}{\Delta x}$$

HOW MUCH WILL IT COST PER EXTRA UNIT TO INCREASE PRODUCTION

FROM  $x$  TO  $x + \Delta x$

# TAKING IDEA FURTHER



## DEFINE MARGINAL COST

$$MC(x) := C'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta C}{\Delta x}$$

NOTICE USE  $\Delta x$  INSTEAD OF  $h$  WHY?



CAN'T EVER HAVE INCREASE OF  $< 1$

UNIT ... SO WHY DO WE USE DERIVATIVE

INSTEAD OF AVERAGE?



WHAT OTHER FUNCTIONS DID WE HAVE?



AVERAGE REVENUE  $\bar{R}(x) = \frac{R(x)}{x}$

MARGINAL REVENUE  $MR(x) = R'(x)$

AVERAGE PROFIT  $\bar{P}(x) = \frac{P(x)}{x}$

MARGINAL PROFIT  $MP(x) = P'(x)$

↳ (AT THE EDGE)



EXAMPLE IF  $C(x) = 6x^2 + 2x + 10$

- (A) FIND  $MC(x)$
- (B) FIND  $MC(10)$  &  $C(10)$
- (C) USE THIS DATA TO ESTIMATE  $C(11)$

(A)  $C'(x) = 12x + 2$

(B)  $MC(10) = 120 + 2 = 122$

$C(10) = 600 + 20 + 10 = 630$

(C)  $MC(10) \approx \frac{C(11) - C(10)}{1}$

$\Rightarrow C(11) \approx C(10) + MC(10) = 752$

THINK OF  $\lim_{\Delta x \rightarrow 0} \frac{C(x + \Delta x) - C(x)}{\Delta x}$

AS STOPPING AT SMALL EST

POSSIBLE VALUE OF  $\Delta x = 1$

(THIS IS DONE IN CALC-FREE ECON)

EX) SAY DEMAND DECREASES

VII

VERY QUICKLY

$$q(p) = 1000 e^{-p}$$

Ⓐ FIND  $R(p)$

Q

$$R(p) = q \cdot p = 1000 p e^{-p}$$

Ⓑ FIND MARGINAL REVENUE WITH RESPECT TO PRICE

Q

$$\begin{aligned} MR(p) = R'(p) &= 1000 (e^{-p} - p e^{-p}) \\ &= 1000 e^{-p} (1-p) \end{aligned}$$

Ⓒ IS THERE A PRICE FOR WHICH MARGINAL PROFIT IS ZERO?

(MAXIMIZED)

Q

$MR(p)$  IS CTS  $\Rightarrow$  CAN USE IVT

$$MR(0) = 1000 e^0 (1-0) = 1000 > 0$$

$$MR(10) = 1000 e^{-10} (1-10) < 0$$

$\hookrightarrow$  OR COULD HAVE GUESSED  $p=1$

WHAT ABOUT CRAZY EQUATION?????