MIDTERM FRIDAY @ 6PM GEOG 200

I

SURVEY ON PIAZZA

STUDENT GUIDE

HTTP://BLOGS.UBC.CA/MATHSTUDENTGUIDE104

STUDY HARD, BUT NOT TOO HARD,

& MAKE SURE YOU GET ENOUGH SLEEP!

[Q] IF $f(x) = \sin(x)$ THEN

(A) $f(x) = f''''(x)$

(B) $f(x) = -f''(x)$

(C) $f'(x) = \cos(x)$

(D) ALL OF THE ABOVE

[Q] $\lim_{h \to 0} \frac{\sin(2x+h) - \sin(2x)}{h}$

(A) $= \cos(x)$

(B) $= \cos(2x)$

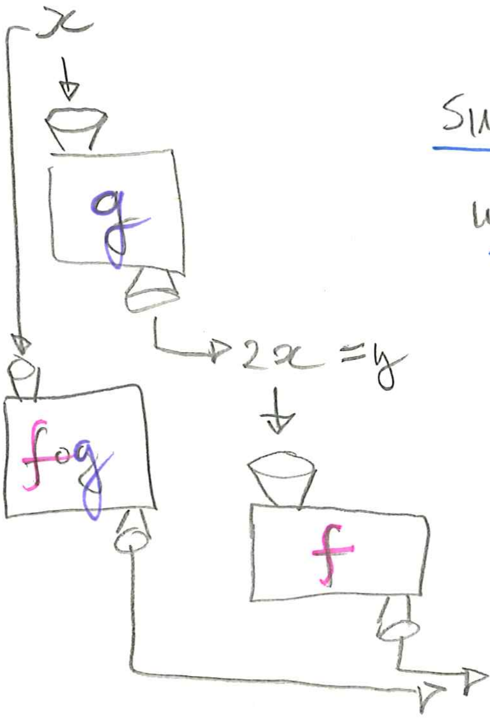
(C) $= 0$

(D) DNE B/C $0/0$ IS NOT DEFINED

WE KNOW $\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$

T/F $\frac{d}{dx}\left(\sin(2x)\right) = \cos(2x)$

WHAT'S GOING ON !?



$\sin(2x) = (f \circ g)(x)$

where

$g(x) = 2x$

$f(y) = \sin(y)$

$\sin(2x) = \sin(y)$

$$x \xrightarrow{g} y \xrightarrow{f} z$$

WE DON'T YET HAVE A DIFFERENTIATION RULE TO HANDLE THIS KIND OF SITUATION

CHAIN RULE

IF g IS DIFF AT a & f IS DIFF AT $g(a)$

THEN $\boxed{(f \circ g)'(a) = f'(g(a)) \cdot g'(a)}$

$$
\begin{tikzcd}
x \arrow[r, bend left, "g"] \arrow[rr, bend right, "f \circ g"'] & g(x) = y \arrow[r, bend left, "f"] & f(y) = z
\end{tikzcd}
$$

CHAIN OF FUNCTIONS

BREAKS UP DERIVATIVE OF COMPLICATED FUNCTION $f \circ g$ INTO DERIVATIVES OF ITS COMPONENTS f & g

OUTER INNER

↳ JUST NEED TO COMPUTE f' & g' & EVALUATE THEM AT THE APPROPRIATE POINTS!

Ex] $h(x) = \sin(2x)$

$= (f \circ g)(x)$

$f(y) = \sin(y)$ DIFF ✓

$g(x) = 2x$ DIFF ✓

CHAIN RULE SAYS

$h'(x) = (f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

$f'(y) = \cos(y)$ $\qquad = \cos(2x) \cdot 2$

$g'(x) = 2$

Ex] $h(x) = (\sin(x))^2$

$= (\sin(x))^2$

$= (f \circ g)(x)$

$f(y) = y^2$ $\qquad f'(y) = 2y$

$g(x) = \sin(x)$ $\qquad g'(x) = \cos(x)$

$h'(x) = (f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

$= 2 \cdot (\sin(x)) \cdot \cos(x)$

Ex $h(x) = \sqrt{e^x}$ (V)

$= \sqrt{e^x} = (f \circ g)(x)$

$f(y) = \sqrt{y}$ $f'(y) = \frac{1}{2\sqrt{y}}$

$g(x) = e^x$ $g'(x) = e^x$

$h'(x) = f'(g(x)) \cdot g'(x)$

$= \frac{1}{2\sqrt{e^x}} \cdot e^x = \frac{e^x}{2\sqrt{e^x}}$

Ex $h(x) = e^{kx}$ $k \in \mathbb{R}$

$= e^{kx}$

$f(y) = e^y$ $f'(y) = e^y$

$g(x) = kx$ $g'(x) = k$

$h'(x) = f'(g(x))\, g'(x) = e^{kx} \cdot k$

Ex] GENERALIZED POWER RULE (VI)

$h(x) = (g(x))^n$ SOME $n \in \mathbb{N}$

$= (g(x))^n$

$f(y) = y^n \qquad f'(y) = ny^{n-1}$

$g(x) = g(x) \qquad g'(x) = g'(x)$

$h'(x) = f'(g(x))\, g'(x)$

$= n \cdot (g(x))^{n-1} \cdot g'(x)$

Ex]

$h(x) = (\sin(2x))^3$

$h'(x) = 3(\sin(2x))^2 (2\cos(2x))$

$= 6 \cdot (\sin(2x))^2 (\cos(2x))$

Ex] $h(x) = \sin\left(\frac{1}{x}\right)$, $\boxed{x \neq 0}$

$h'(x) = \cos\left(\frac{1}{x}\right) \cdot \left(\frac{-1}{x^2}\right)$

VII

Ex] $h(x) = \cos(e^{\sqrt{x}})$

DO CHAIN RULE TWICE!

$= \cos(e^{\sqrt{x}})$

$f(y) = \cos(y) \qquad f'(y) = -\sin(y)$

$g(x) = e^{\sqrt{x}} \qquad g'(x) =$ CHAIN RULE

$g(x) = e^{\sqrt{x}}$

$f(y) = e^{y} \qquad f'(y) = e^{y}$

$g(x) = \sqrt{x} \qquad g'(x) = \frac{1}{2\sqrt{x}}$

$g'(x) = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$h'(x) = -\sin(e^{\sqrt{x}}) \cdot \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

START WITH OUTERMOST FUNCTION

& USE CHAIN RULE LAYER BY

LAYER

CHAIN RULE

VIII

IF g DIFF AT a

& f DIFF AT $g(a)$

THEN $f \circ g$ IS DIFF AT a

& $\boxed{(f \circ g)'(a) = f'(g(a)) \cdot g'(a)}$

Q: HOW WOULD YOU GO ABOUT CHECKING IF FORMULA IS TRUE?

$$\lim_{h \to 0} \frac{(f \circ g)(a+h) - (f \circ g)(a)}{h}$$

$$= \lim_{h \to 0} \frac{f(g(a+h)) - f(g(a))}{h}$$

WHERE DOES $g'(a)$ COME FROM?

$$= \lim_{h \to 0} \underbrace{\left(\frac{f(g(a+h)) - f(g(a))}{g(a+h) - g(a)}\right)}_{\text{"} f'(g(a)) \text{"}} \underbrace{\left(\frac{g(a+h) - g(a)}{h}\right)}_{\text{"} g'(a) \text{"}} \ldots$$