- MIDTERM I GRADES ON PIAZZA (I)

WILL BE HANDED BACK @ END OF CLASS (& SAY FEW WORDS)

- FILL OUT ANONYMOUS SURVEY

LAST TIME

CHAIN RULE (MASTER TOOL)

IF g IS DIFF AT a

& f IS DIFF AT $g(a)$

THEN $f \circ g$ IS DIFF AT a

AND MEMORIZE

$$(f \circ g)'(a) = f'(g(a)) \cdot g'(a)$$

[Q]

IF α & β ARE DIFF FUNCTIONS & $f = \alpha \circ \beta$ THEN $f'(2) =$

(a) $\alpha'(2) \circ \beta'(2)$ (b) $\alpha'(2) \cdot \beta'(2)$

(c) $\alpha'(\beta(2))\,\beta'(2)$ (d) $\alpha'(\beta(x))\,\beta'(2)$

[Q] SUPPOSE RADIUS $r(t)$ OF A CIRCLE CHANGES OVER TIME

THEN ITS AREA AT TIME "t" IS GIVEN BY $A(t) = \pi (r(t))^2$.

THE RATE OF CHANGE IN AREA WITH RESPECT TO TIME $\frac{d}{dt}(A(t))$

(a) $= 2\pi r(t)$

(b) $= 2\pi r(t) + \frac{d}{dt}(r(t))$

(c) $= 2\pi r(t) \frac{d}{dt}(r(t))$

(d) NOT ENOUGH INFO.

RECALL: DOMAIN RANGE

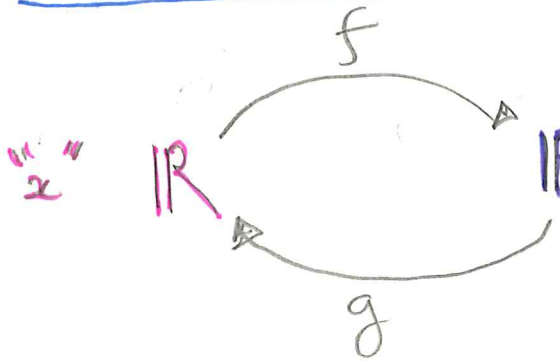
Q $D \xrightarrow{f} R$ IS INVERTIBLE IF ?

$\Rightarrow \exists!$ ONE-TO-ONE $R \xrightarrow{g} D$ S.T. ? Q

$g(f(x)) = x \wedge f(g(y)) = y$

PROPERTIES OF f & g INTERTWINE (III)

FAVOURITE EXAMPLE



$f(x) = e^x$

$g(y) = \ln(y)$

$$f(g(y)) = y \rightsquigarrow e^{\ln(y)} = y$$

$$g(f(x)) = x \rightsquigarrow \ln(e^x) = x$$

$$e^{a_1 + a_2} = e^{a_1} e^{a_2}$$

KNOW

$$\ln(b_1) + \ln(b_2) = \ln(b_1 b_2)$$

DISCOVER

$$e^{\ln(b_1) + \ln(b_2)} = e^{\ln(b_1)}\, e^{\ln(b_2)} = b_1 b_2$$

$$\Rightarrow \ln\left(e^{\ln(b_1) + \ln(b_2)}\right) = \ln(b_1 b_2)$$

IV

KNOW $\frac{d}{dx}[e^x] = e^x$

Q: CAN WE USE THIS TO FIND $\frac{d}{dy}[\ln(y)]$?

CONSIDER $h(x) = x \Rightarrow \boxed{h'(x) = 1}$

BUT THINK OF IT AS

$h(x) = (g \circ f)(x) = \ln(e^x) = x$

USE CHAIN RULE

$h'(x) = g'(f(x)) \cdot f'(x)$

$= g'(e^x) \cdot e^x$

REMEMBER $f(x) = e^x = y$

$\boxed{= g'(y) \cdot y}$

Q WHY IS THIS ALRIGHT

COMBINE

$\Rightarrow g'(y) \cdot y = 1 \Rightarrow g'(y) = \frac{1}{y}$

$\Rightarrow \frac{d}{dy}[g(y)] = \frac{1}{y} \Rightarrow \frac{d}{dy}[\ln(y)] = \frac{1}{y}$

CONCLUDE

V

$$\frac{d}{dx}\left[e^x\right] = e^x \leftrightarrow \frac{d}{dy}\left[\ln(y)\right] = \frac{1}{y}$$

Q: WHY AM I USING SYMBOLS x & y?

HW: CAN THIS PROCEDURE BE APPLIED TO OTHER FUNCTIONS & THEIR INVERSES?

Q: LET g BE A FUNCTION WITH $g(x) > 0$. CAN WE HANDLE

$$\frac{d}{dx}\left[\ln(g(x))\right] ?$$

EXAMPLE?

CHAIN RULE

$$\frac{d}{dx}\left((\ln \circ g)(x)\right) = \frac{1}{(g(x))} \cdot g'(x)$$

NOT SO CLEAR BUT KNOWING

$$\frac{d}{dy}\left[\ln(y)\right]$$ IS GREAT PROGRESS

VI

COOL TRICK

LOGARITHMIC DIFFERENTIATION

LET $f(x) = x^x$, CAN YOU FIND $f'(x)$?

TAKE ln! $\ln(f(x)) = \ln(x^x) = x\ln(x)$

LOG RULE

CHAIN RULE $\frac{d}{dx}$

PRODUCT RULE $\frac{d}{dx}$

$$\frac{f'(x)}{f(x)} = 1\cdot\ln(x) + x\cdot\frac{1}{x} = \ln(x)+1$$

SOLVE FOR $f'(x)$

$$\Rightarrow f'(x) = (\ln(x)+1)\cdot f(x)$$

$$= (\ln(x)+1)\cdot x^x$$

READ SECTION 3.8

DON'T FORGET WEBWORK

OK TALK ABOUT MIDTERM

VII

#1 BASIC LIMITS

↳ FACTOR + CANCEL / MULT CONJUGATE

#2 • TRIPLE PRODUCT RULE

$$(x e^x)(\cos(x))$$

- $\dfrac{e^{6x} \cdots e^{10x}}{e^{2} \cdots e^{5x}}$ ⇝ POWER LAWS

NO QUOTIENT

#3 JUST LIKE W-CLASS QUIZ

#4

① $f(x) = \begin{cases} ax^2 + x & x \le -1 \\ (x+a)^2 & x > -1 \end{cases}$

② $f\left(-\frac{1}{2}\right) = \frac{1}{4}$

① $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1) = a - 1$

$\lim_{x \to -1^+} f(x) = (-1+a)^2$

⇝ $a = 1$ OR $a = 2$

② $\left(-\frac{1}{2} + a\right)^2 = \frac{1}{4}$ ⇝ $a = 1$ OR $a = 1$

$\Rightarrow$ $\boxed{a = 1}$

VIII

#5 BUSINESS PROBLEM

GIVEN $p(q)$ & $C(q)$

(a) $R(q) = p(q) \cdot q$

(b) $q = 70$ ↑ WILL REVENUE ↑ ↓ ?

$R'(q) \rightsquigarrow R'(70) > 0 \Rightarrow$ INCREASE

(c) $C(q) = R(q)$ BREAK EVEN

$\rightsquigarrow 0 = R(q) - C(q)$

$\rightsquigarrow$ USE IVT 90 & 100

#6 (a) $\frac{d}{dx}[x|x|]$ AT $x = 0$, JUST TAKE DEF $\lim$

(b) TAKE DERIVATIVE

$\rightsquigarrow$ LINE EQUATION

- ONLY 1 in / 2 SPECIFIC Q'S
- REGRADING $\rightsquigarrow$ DON'T INITIATE UNLESS THINK MORE THAN 5 POINTS MISSING