- MIDTERM SOLUTIONS ONLINE (COMMON COURSE WEBPAGE)
- ANONYMOUS SURVEY FILL OUT & RETURN AT THE END OF CLASS

I

[T/F] IF g IS A DIFF FUNCTION WHICH IS NEVER EQUAL TO ZERO, i.e., $g(x)<0$ OR $g(x)>0$ THEN

$$\left(\frac{\frac{d}{dx}[g(x)]}{g(x)}\right) = h(x)$$

IS NOT THE DERIVATIVE OF ANY FUNCTION.

[T/F] THE EQUATION

$x^2+y^2=1$ DEFINES A FUNCTION.

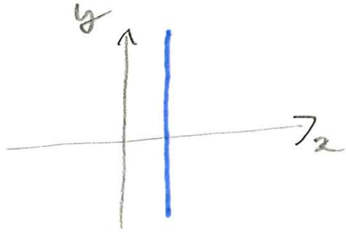
~~Q: WHAT WERE THE ONLY PROPERTIES OF $f(x)=e^x$ & $g(y)=\ln(y)$ THAT WE USED TO COMPUTE $\frac{d}{dy}[\ln(y)]$?~~

IN GENERAL, EQUATIONS DEFINE SUBSETS OF THE PLANE WHICH CORRESPOND TO ALL OF THEIR SOLUTIONS

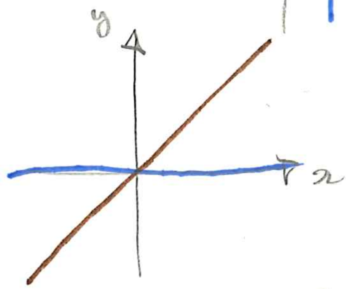
II

EX

$x = 1$

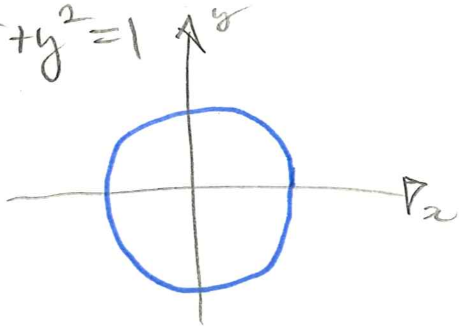


$y = 0$

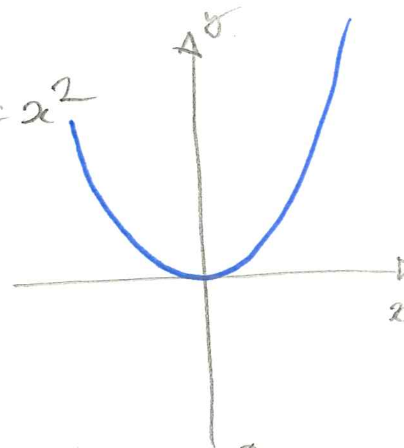


$y = x$

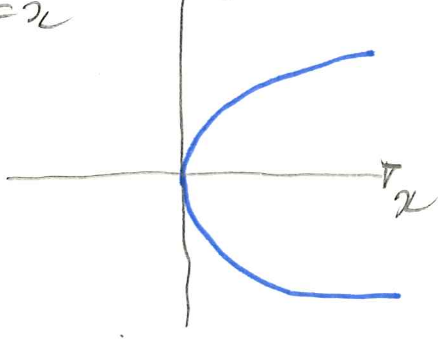
$x^2 + y^2 = 1$



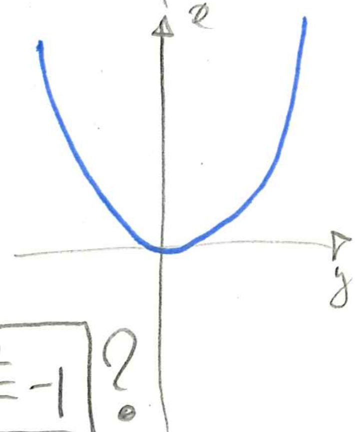
$y = x^2$



$y^2 = x$

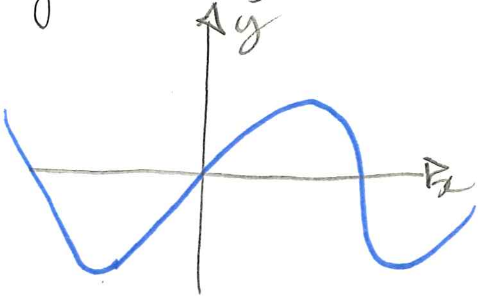


OR

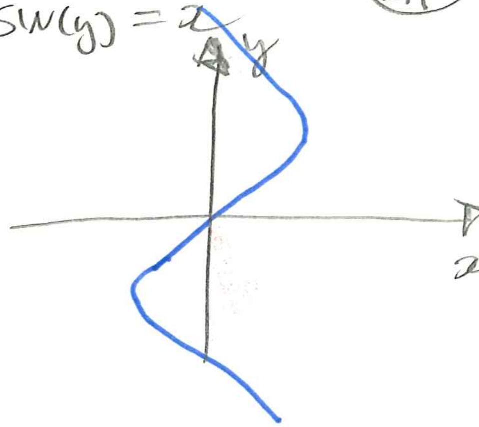


$\boxed{x^2 = -1}$?

$y = \sin(x)$

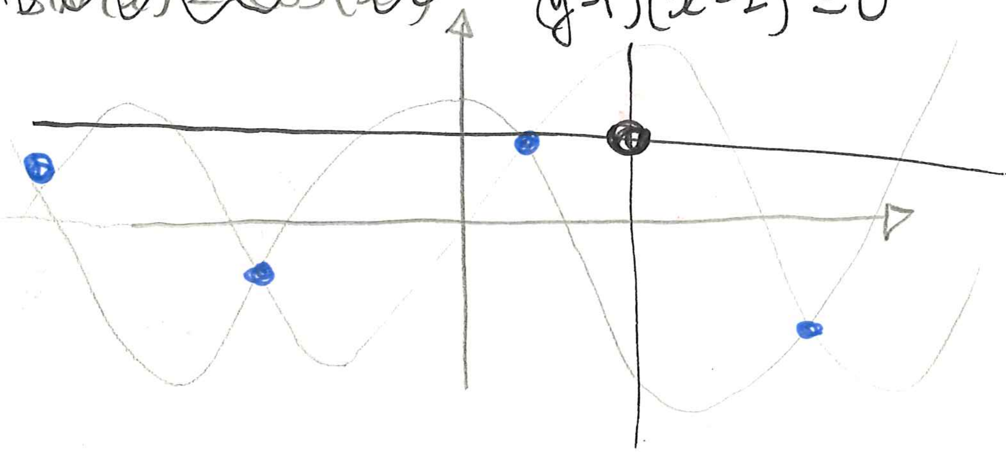


$\sin(y) = x$



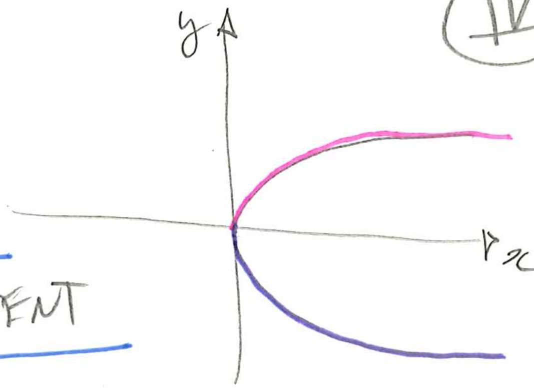
$\sin(x) = \cos(xy)$

$(y-1)(x-2) = 0$



THESE SOLUTION SETS CAN MAKE CRAZY PATTERNS & MAY NOT REPRESENT FUNCTIONS... (OFTEN) BUT THEY CAN BE DESCRIBED BIT BY BIT USING MANY IMPLICITLY DEFINED FUNCTIONS

$y^2 = x$ (IV)



IF WE THINK OF
x AS INDEPENDENT
& y AS DEPENDENT
i.e. $(y(x))^2 = x$

$\Rightarrow \begin{cases} y(x) = \sqrt{x} \\ y(x) = -\sqrt{x} \end{cases}$ ARE THE TWO "HIDDEN" FUNCTIONS

IF WE THINK OF y AS INDEPENDENT
& x AS DEPENDENT, IE $y^2 = x(y)$
THEN IT IS A FUNCTION

WE'VE SEEN THIS BEFORE

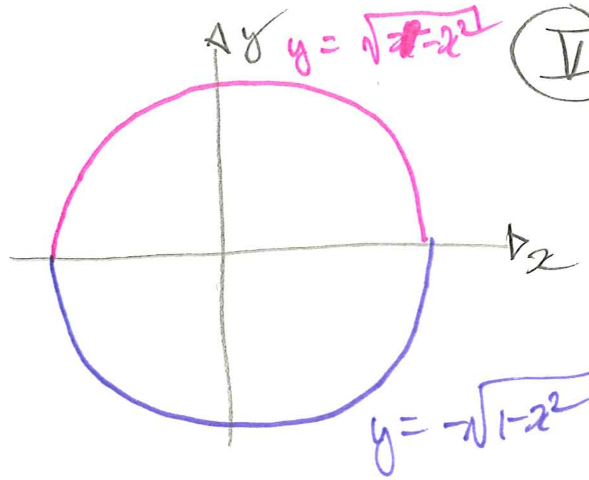
$p = aq + b$ a & b CONSTANTS

$\Rightarrow p(q) = aq + b$ q INDEP p DEP

$\frac{p-b}{a} = q(p)$ q DEP p INDEP

EVERYTHING IS A FUNCTION OF EVERYTHING

$$\boxed{x^2 + y^2 = 1}$$



WANT

y DEPENDENT

x INDEPENDENT

IE $x^2 + (y(x))^2 = 1$ WHAT IS $y'(x)$

COULD DO BOTH BRANCHES SEPARATELY

& COMPARE ... OR USE

IMPLICIT DIFFERENTIATION

HIT BOTH SIDES OF EQUATION WITH

"OPERATOR" $\frac{d}{dx}\left(x^2 + (y(x))^2\right) = \frac{d}{dx}(1)$

$$\frac{d}{dx}\left[x^2\right] + \frac{d}{dx}\left[(y(x))^2\right] = \frac{d}{dx}[1]$$

$$2x + 2\,y(x)\,\frac{d}{dx}[y(x)] = 0$$

SOLVE $\boxed{\frac{d}{dx}[y(x)] = \frac{-x}{y(x)}}$

Q WHAT PROPERTIES OF OUR EQUATION DID WE USE? (VI)

WHEN WE WRITE $\frac{d}{dx}\left[y(x)\right]$ WE ARE **ASSUMING** THAT THE IMPLICITLY DEFINED EQUATION (IN OUR CASE EITHER $y(x)=\sqrt{1-x^2}$ OR $y(x)=-\sqrt{1-x^2}$) IS DIFFERENTIABLE AT x!!!

IN OUR CASE THE TWO FUNCTIONS ARE DIFF EXCEPT AT $x=\pm 1$ OR WHERE $y(x)=0$

(NOTICE WE HAD TO DIVIDE BY $y(x)$...)

SHOOT FIRST, ASK QUESTIONS LATER (MAKE SURE WHAT YOU DID MAKES SENSE)

WESTERN

SANITY CHECK

$y(x) = \sqrt{1-x^2}$ CHAIN RULE

$\rightsquigarrow y'(x) = \frac{1}{2}(1-x^2)^{-1/2} \cdot -2x$

$= \frac{-x}{(1-x^2)^{1/2}} = \frac{-x}{y(x)}$

$y(x) = -\sqrt{1-x^2} \rightsquigarrow y'(x) = \frac{-1}{2}(1-x^2)^{-1/2} \cdot -2x$

NOT SAME $y(x)$

$= \frac{x}{\sqrt{1-x^2}} = \frac{-x}{y(x)}$

TWO FOR ONE!

SOMETIMES HARD TO SOLVE EXPLICITLY (IMPOSSIBLE)

EX $\boxed{y^2 - y^3 = x^2}$

CHOOSE DEPENDENT & INDEP VARIABLE

$\boxed{(y(x))^2 - (y(x))^3 = x^2}$

TAKE $\frac{d}{dx}$ OF BOTH SIDES

USING CHAIN RULE WHEN APPROPRIATE

VIII

$$2\,y(x)\,\frac{d}{dx}[y(x)] + 3(y(x))^2\,\frac{d}{dx}[y(x)]$$

$$\|$$

$$2x$$

SOLVE FOR $\frac{d}{dx}[y(x)]$

$$\frac{d}{dx}[y(x)] \cdot \left(2y(x) - 3(y(x))^2\right) = 2x$$

$$\Rightarrow \boxed{\frac{d}{dx}[y(x)] = \frac{2x}{2y(x) - 3(y(x))^2}}$$

ASK QUESTIONS!

$y(x) = 0$ BAD

OK WHAT ABOUT PLACES WHERE $y(x) \neq 0$

$2\,y(x) - 3\,(y(x))^2 = 0$ BAD

$y(x)\left(2 - 3\,y(x)\right) = 0$

$\boxed{\frac{2}{3} = y(x)}$ BAD

USUALLY PUT RESTRICTIONS ON DOMAIN FOR DIFFERENTIABILITY

$\boxed{y(x)=0} \rightsquigarrow y(x)^2 - y(x)^3 = x^2$

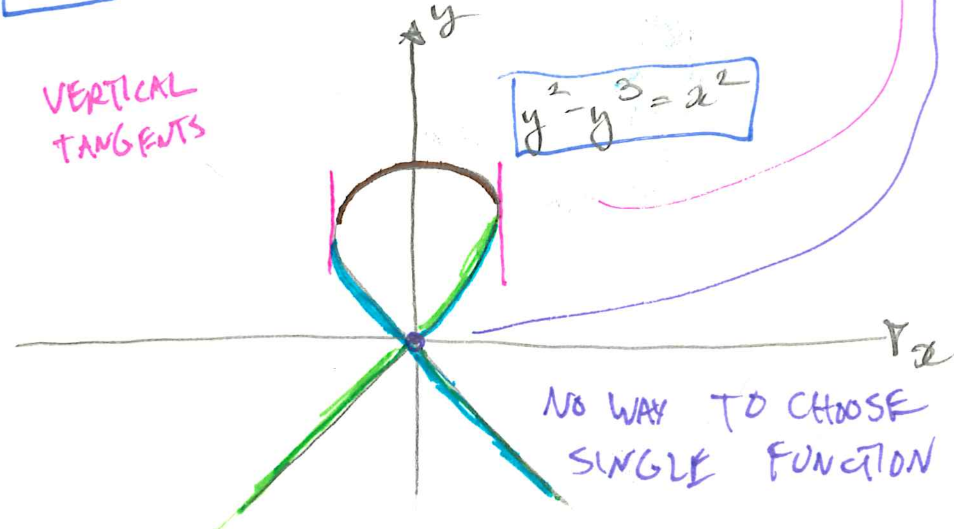
$\Rightarrow 0 - 0 = x^2 \Rightarrow \boxed{x=0}$

$\boxed{y(x)=\frac{2}{3}} \rightsquigarrow \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3 = x^2$

$\Rightarrow \boxed{\pm\sqrt{\frac{4}{9} - \frac{8}{27}} = x}$

FORMULA VALID EXCEPT FOR

THREE BAD VALUES OF x



HOMEWORK FOR FRIDAY: USE IMPLICIT DIFFERENTIATION TO FIND $\frac{d}{dx}\left[x^{\frac{n}{m}}\right]$ WHERE n, m ARE POSITIVE INTEGERS.