LECTURES ARE

TOO FAST ≈ 10%
TOO SLOW ≈ 5%
ABOUT RIGHT ≈ 85%

SLOW ME DOWN

MOST PEOPLE WORK ON THEIR OWN

↳ FIND A WORK BUDDY!

WANT MORE EXAMPLES

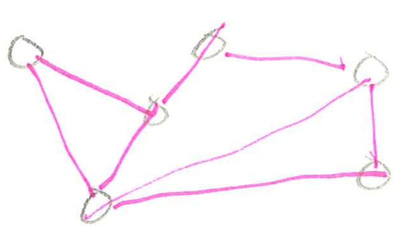
WANT MORE QUIZZES

↳ 1 LONG PER EXAM

↳ MORE SHORT ANSWER ONES

PHILOSOPHY OF THIS SECTION (EVERYTHING FOR A REASON)

IT IS EASIER TO MEMORIZE & APPLY CONCEPTS WHEN YOU HAVE SOME UNDERSTANDING IDEA ABOUT HOW THEY ALL RELATE TO EACH OTHER



BUILD NETWORK OF KNOWLEDGE
VS
ISOLATED FACTS

[Q] WHICH OF THE FOLLOWING FUNCTIONS ARE IMPLICITLY DEFINED BY THE EQUATION $x^2 + y^2 = 1$? II

(A) $y(x) = \sqrt{x^2 - 1}$

(B) $y(x) = \sqrt{1 - x^2}$

(C) $x(y) = -\sqrt{y^2 - 1}$

(D) $x(y) = -\sqrt{1 - y^2}$

HW [Q] HOW DID YOU USE IMPLICIT DIFFERENTIATION TO COMPUTE

$$\frac{d}{dx}\left[x^{\frac{n}{m}}\right]$$

USING YOUR KNOWLEDGE THAT

$$\frac{d}{dx}\left[x^{\ell}\right] = \ell x^{\ell - 1}$$

FOR ALL INTEGERS ℓ ?

$$y(x) = x^{\frac{n}{m}}$$

$$\Rightarrow (y(x))^m = \left(x^{\frac{n}{m}}\right)^m \Rightarrow (y(x))^m = x^n$$

...

PLAY A BIT MORE WITH IMPLICIT DIFFERENTIATION THEN GET BACK DOWN TO OUR MAIN TASK: UNDERSTANDING SHAPE OF GRAPH OFF DIFF FUNCTIONS

III

RECALL $x \in \mathbb{R}$ $\ln(e^x) = x$

$y \in \mathbb{R}_{>0}$ $e^{\ln(y)} = y$

[Q] WHAT IS $\frac{d}{dz}(\pi^z)$?

↳ REWRITE π^z AS SOMETHING THAT LOOKS MORE COMPLICATED BUT THAT WE UNDERSTAND BETTER

$$f(z) = \pi^z = e^{\ln(\pi^z)} = e^{z\ln(\pi)}$$

OK!?

$$\Rightarrow f'(z) = \ln(\pi) \cdot e^{z\ln(\pi)}$$

$$= \ln(\pi)\, e^{\ln(\pi^z)}$$

$$= \ln(\pi) \cdot \pi^z$$

IN GENERAL $\alpha(x) = b^x \Rightarrow \alpha'(x) = \ln(b)\, b^x$

[Q] WHAT IS $\frac{d}{dz}[z^{\pi}]$? (IV)

$$h(z) = z^{\pi} = e^{\ln(z^{\pi})} = e^{\pi \ln(z)}$$

OK!?

CAN WE COMPUTE THIS DERIVATIVE?

$$= e^{(\pi \ln(z))}$$

$$\Rightarrow h'(z) = e^{(\pi \ln(z))} \cdot (\pi \ln(z))'$$

$$= e^{\pi \ln(z)} \cdot \frac{\pi}{z}$$

WORKS IN GENERAL

$$= z^{\pi} \cdot \frac{\pi}{z} = \pi \cdot z^{\pi - 1}$$

<u>RECALL</u> IF $b > 0$ & $b \neq 1$ $f(x) = b^{x}$

IS ONE-TO-ONE WITH INVERSE

$$g(x) = \mathrm{LOG}_b(y)$$

CAN THINK OF INVERSE FUNCTION RELATIONSHIP AS (TAKE $\mathrm{LOG}_b(\,)$)

$$y = b^{x} \Leftrightarrow \mathrm{LOG}_b(y) = \mathrm{LOG}_b(b^{x}) = x$$

IN $y = b^x$ THINK OF (V)

y AS INDEPENDENT VARIABLE
x AS DEPENDENT VARIABLE

$\leadsto y = b^{x(y)}$

IMPLICIT DIFFERENTIATION: HIT BOTH SIDES OF EQUATION WITH $\frac{d}{dy}[\ ]$

$$\frac{d}{dy}[y] = \frac{d}{dy}\left[b^{(x(y))}\right]$$ CHAIN RULE

$$1 = \left(\ln(b) \cdot b^{(x(y))}\right) \cdot \frac{d}{dy}[x(y)]$$

$$\frac{1}{\ln(b) \cdot b^{x(y)}} = \frac{d}{dy}[x(y)]$$

$$\frac{1}{\ln(b) \cdot y} = \frac{d}{dy}[x(y)] = \frac{d}{dy}[\log_b(y)]$$

NOTICE y NEEDS TO BE NON-ZERO

OK LET'S DO EXAMPLES

[Q] $x^3 + y^3 = 2xy$ (VI)

FIND EQUATION OF TANGENT LINE

@ $(1,1)$

$\hookrightarrow$ $1^3 + 1^3 = 2(1)(1)$ ✓

CHOOSE DEPENDENT VS INDEP VARIABLES

$$x^3 + (y(x))^3 = 2 \cdot x \cdot y(x)$$

TAKE $\frac{d}{dx}$ OF BOTH SIDES!

$$\frac{d}{dx}\left[x^3 + (y(x))^3\right] = \frac{d}{dx}\left[2x \cdot y(x)\right]$$

$$3x^2 + 3(y(x))^2 \frac{d}{dx}[y(x)]$$

$$\parallel$$

$$2y(x) + 2x\frac{d}{dx}[y(x)]$$

$$3(y(x))^2 \frac{d}{dx}(y(x)) - 2x\frac{d}{dx}[y(x)] = 2y(x) - 3x^2$$

$$\boxed{\frac{d}{dx}[y(x)] = \frac{2y(x) - 3x^2}{3(y(x))^2 - 2x}}$$

EVALUATE AT $x=1$ $y=1$

$$y'(1) = \frac{2\cdot(1) - 3\cdot(1)^2}{3(1)^2 - 2(1)} = \frac{-1}{1} = -1$$

TANGENT LINE

$\ell(x) = mx + b$ $\boxed{m=-1}$

$\ell(x) = -x + b$

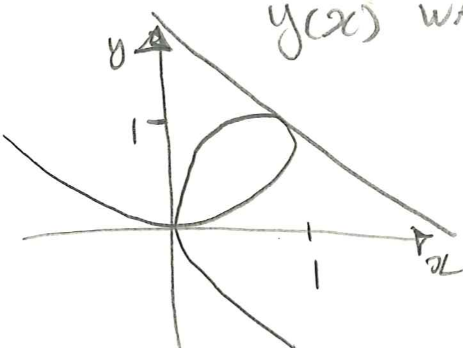
KNOW $(1,1)$ IS ON LINE

$\Rightarrow 1 = \ell(1) = -1 + b$

$\Rightarrow 2 = b$

$\Rightarrow \boxed{\ell(x) = -x + 2}$ IS DESIRED EQUATION

NOTICED NEVER FOUND OUT WHAT $y(x)$ WAS EQUAL TO!



CHALLENGE OF DESCARTE TO FERMAT!

VIII

SUMMARY

- $$\frac{d}{dx}(b^x) = \ln(b) \cdot b^x$$

- $$\frac{d}{dy}(\log_b(y)) = \frac{1}{\ln(b) \cdot y}$$

GIVEN EQUATION WHERE HARD TO SOLVE FOR x OR y CAN DIFFERENTIATE IMPLICITLY

1. CHOOSE DEP $\square$ VS INDEP Δ VARIABLE

2. TAKE $\frac{d}{d\square}[\quad]$ OF BOTH SIDES

3. SOLVE FOR $\frac{d}{d\square}(\Delta(\square))$

4. CHECK FOR DIVISION BY ZERO !

FACT FOR WEBWORK

TWO LINES ARE ORTHOGONAL IF THE PRODUCT OF THEIR SLOPES IS $\boxed{-1}$

TWO CURVES ARE ORTHOGONAL IF THEIR TANGENT LINES ARE ORTHOGONAL WHENEVER THEY INTERSECT.