MIDTERM FRIDAY OCTOBER 3RD @ 6PM

I

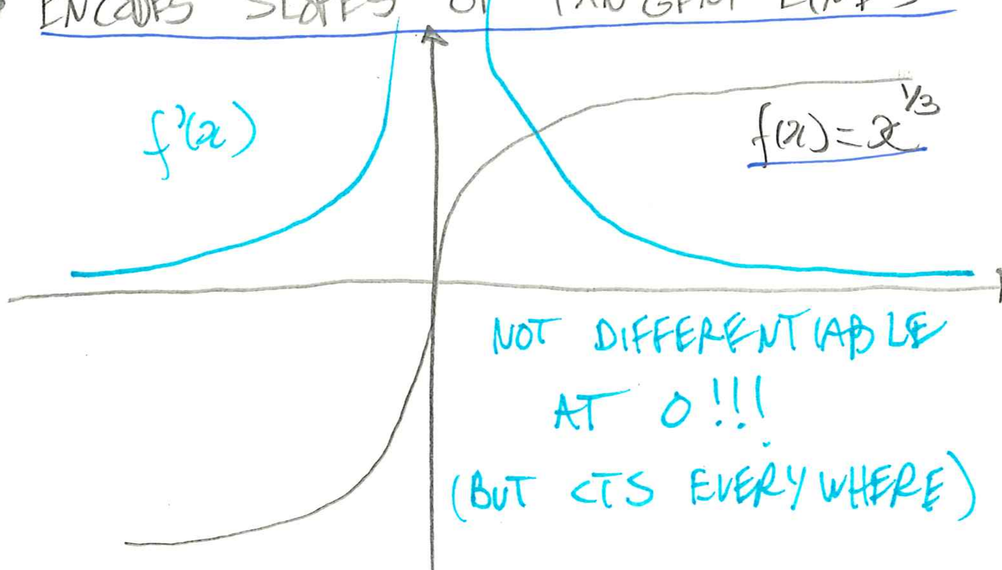
- PRACTISE MIDTERM + SOLUTIONS
- PAST FINALS + SOLUTIONS ON MATH WIKI
- MATH 184 WORKSHOP PROBLEMS + SOLUTIONS

WEBSITE

POSTED WEBWORK 1, #10 SOLUTION ON PIAZZA

LAST TIME

- IF $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ EXISTS WE CALL IT THE DERIVATIVE OF f AT a, & WRITE IT AS $f'(a)$
- THINK OF $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ AS A FUNCTION WHEREVER IT EXISTS
- ENCODES SLOPES OF TANGENT LINES



NOT DIFFERENTIABLE AT 0!!!
(BUT CTS EVERYWHERE)

Q SUPPOSE $f'(a)$ EXISTS, II

THEN $\lim_{x \to a} f(x)$:

(A) MUST EXIST BUT NOT ENOUGH INFORMATION TO DETERMINE ITS EXACT VALUE

(B) EQUALS $f(a)$

(C) EQUALS $f'(a)$

(D) MIGHT NOT EXIST

DIFF $\Rightarrow$ CTS

NOT CTS $\Rightarrow$ NOT DIFF

Q A SLOW TRAIN CHUGS ALONG STRAIGHT TRACK. THE DISTANCE IT HAS TRAVELLED AFTER t HOURS IS GIVEN BY A FUNCTION $d(t)$. AN ENGINEER IS WALKING ON TOP OF A BOXCAR AT THE RATE OF 3 MI/HOUR IN SAME DIRECTION AS TRAIN. HER SPEED RELATIVE TO THE GROUND IS:

(A) $d(t)+3$

(B) $d'(t)+3$

(C) $d(t)-3$

(D) $d'(t)-3$

SHOULD BE ABLE TO INTERPRET DERIVATIVES AS RATES OF CHANGE!

III

TODAY: FIND EASY RULES FOR DIFFERENTIATING LARGE CLASS OF FUNCTIONS FORMED BY ADDING, MULTIPLYING & DIVIDING EASY FUNCTIONS.

- $f(x) = c \Rightarrow f'(x) = 0$
- $g(x) = x \Rightarrow g'(x) = 1$

WHAT HAPPENS WHEN WE ADD?

$l(x) = f(x) + g(x)$

$$\lim_{h \to 0} \frac{l(a+h) - l(a)}{h}$$

$$= \lim_{h \to 0} \frac{(f(a+h) + g(a+h)) - (f(a) + g(a))}{h}$$

$$= \lim_{h \to 0} \frac{(c + (a+h)) - (c + a)}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = 1$$

$l'(x) = 1 = f'(x) + g'(x)$

$\boxed{Q}$ IS THIS AN ACCIDENT?

GO BACK TO OUR COMPUTATION

IV

$$l(x) = f(x) + g(x)$$

DIFFERENTIABLE AT a

$$\lim_{h\to 0} \frac{l(a+h) - l(a)}{h} = \lim_{h\to 0} \frac{(f(a+h)+g(a+h)) - (f(a)+g(a))}{h}$$

$$= \lim_{h\to 0}\left(\frac{f(a+h) - f(a) + g(a+h) - g(a)}{h} \right)$$

$$= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} + \frac{g(a+h) - g(a)}{h}$$

$$= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} + \lim_{h\to 0} \frac{g(a+h) - g(a)}{h}$$

$\boxed{Q}$ WHY CAN I DO THIS?

$$= f'(a) + g'(a)$$

$\boxed{EX}$

$$K(x) = 2x + x^3$$
$$= \underset{f(x)}{x} + \underset{g(x)}{x} + \underset{h(x)}{x^3}$$

$$K'(x) = f'(x) + g'(x) + h'(x)$$
$$= 1 + 1 + 3x^2 = 2 + 3x^2$$

WHAT HAPPENS WHEN WE MULTIPLY? (V)

$f(x) = c$, $g(x) = x$

$\ell(x) = f(x) \cdot g(x)$

$$\lim_{h \to 0} \frac{f(a+h)\,g(a+h) - f(a)\,g(a)}{h}$$

$$= \lim_{h \to 0} \frac{c \cdot (a+h) - c \cdot a}{h}$$

$$= \lim_{h \to 0} \frac{ca + ch - ca}{h}$$

$$= \lim_{h \to 0} \frac{ch}{h} = c \lim_{h \to 0} \frac{h}{h} = c \cdot 1 = c$$

NOT SO CLEAR... $= c \cdot g'(x)$

$$\lim_{h \to 0} \frac{c \cdot g(a+h) - c \cdot g(a)}{h}$$

$$= \lim_{h \to 0} c \cdot \left(\frac{g(a+h) - g(a)}{h} \right)$$

$$= c \cdot \lim_{h \to 0} \frac{g(a+h) - g(a)}{h} = c \cdot g'(a)$$

CONSTANT MULTIPLE RULE

WHAT WOULD YOU EXPLORE NEXT?

$\ell(x) = x^2 \rightsquigarrow \ell'(x) = 2x$

$\ell(x) = x^3 \rightsquigarrow \ell'(x) = 3x^2$

$\ell(x) = x^4 \rightsquigarrow \ell'(x) = 4x^3$

BY LIMIT DEFINITION

GUESS $\ell(x) = x^n \rightsquigarrow \ell'(x) = n x^{(n-1)}$

$\Rightarrow$ $\boxed{\text{TRUE!}}$ SEE PROOF IN BOOK

HAVE WE FIGURED OUT EVERYTHING WE COULD ABOUT PRODUCTS?

$f(x) = x^5 \rightsquigarrow f'(x) = 5x^4$

TRY SPLITTING INTO PRODUCTS

$f(x) = x^1 x^4 = p(x) \cdot g(x)$

$p'(x) = 1 \quad g'(x) = 4x^3 \rightsquigarrow 5x^4$

$f(x) = x^2 x^3 = p(x) \cdot g(x)$

$p'(x) = 2x \quad g'(x) = 3x^2 \rightsquigarrow 5x^4$

STILL NOT SO CLEAR?

VII

$f(x) = x^{24} \rightsquigarrow f'(x) = 24x^{23}$

$f(x) = p(x)\,g(x) = x^{5}\,x^{19}$

$\left.\begin{array}{ll} p(x) = x^{5} & g(x) = x^{19} \\ p'(x) = 5x^{4} & g'(x) = 19x^{18} \end{array}\right\} \rightsquigarrow 24x^{23}$

PRODUCT RULE $f'(x) = p'(x)g(x) + p(x)g'(x)$

WHENEVER $f(x) = p(x)\cdot g(x)$ & BOTH

$p(x)$ & $g(x)$ ARE DIFFERENTIABLE

EX $f(x) = \underbrace{(3x^2 + x^3)}_{p(x)} \cdot \underbrace{(\sqrt{x} + 1)}_{g(x)}$

COMPUTE $f'(1)$

IS f DIFFERENTIABLE AT 1

$p'(x) = 6x + 3x^2$

$g'(x) = \frac{1}{\sqrt{x}} + 0$ (SAW IN 3.1)

$f'(1) = p'(1)\,g(1) + p(1)\,g'(1)$

$= 9 \cdot 2 + 4 \cdot 1 = 22$

LAST BUT NOT LEAST: QUOTIENTS VIII

IF U & V ARE DIFFERENTIABLE AT a WITH $V(a) \neq 0$ &

$$g(x) = \frac{u(x)}{v(x)}$$ THEN

$$g'(x) = \frac{u'(x)\,v(x) - u(x)\,v'(x)}{(v(x))^2}$$

"UP PRIME DOWN MINUS UPP DOWN PRIME OVER DOWN SQUARED"

EX)

$$g(x) = \frac{4x}{x^2+1}$$

$u(x) = 4x \qquad u'(x) = 4$

$v(x) = x^2+1 \qquad v'(x) = 2x$

$$\Rightarrow g'(x) = \frac{4(x^2+1) - (4x)(2x)}{(x^2+1)^2}$$

↳ BOTTOM NEVER VANISHES.

WE CAN NOW HANDLE DERIVATIVES OF ALL POLYNOMIALS & QUOTIENTS OF POLYNOMIALS! (ONLY USES CONSTANT & IDENTITY)

LOOK AT 3.3 & 3.4

A $\Rightarrow$ B IS SAME AS SAYING

(NOT B) $\Rightarrow$ (NOT A)

I AM NOT SUNBURNT $\Rightarrow$ I DID NOT STAND NAKED IN THE SUN FOR HOURS

$(n+1)$ IS NOT ODD $\Rightarrow$ n IS NOT EVEN

RECALL: IF $f(x)$ IS A POLYNOMIAL, THEN $f(x)$ IS CTS

i.e, $f(x)$ IS A POLYNOMIAL $\Rightarrow$ $f(x)$ IS CTS

Q WHICH OF THE FOLLOWING IS T?

(A) IF $f(x)$ IS NOT CTS THEN $f(x)$ IS NOT A POLY

(B) IF $f(x)$ IS CTS THEN IT IS A POLY

(C) IF $f(x)$ IS NOT A POLY THEN IT IS CTS

OUR THM HAS A MYSTERIOUS COUNTERPART

$$\boxed{\text{IF } f \text{ IS NOT CTS AT } a \text{ THEN } f \text{ IS NOT DIFFERENTIABLE AT } a}$$

GET A HEAD START: DO THE WHOLE WEBWORK!