IT'S THE END OF THE WEEK (I)

→ TAKE A LOOK @ THE WEEKLY LEARNING GOALS !

CONFERENCE FOR LEARNING AND STUDENT SUCCESS

→ SATURDAY OCT 18 (MY. SCIENCE. UBC. CA)

LAST TIME

PRICE ELASTICITY OF DEMAND

$$\boxed{\varepsilon(p) := \frac{p}{q}\frac{dq}{dp}} \leftarrow \text{SHORT HAND FOR} \quad \frac{p}{q(p)}\frac{d}{dp}\left[q(p)\right]$$

• THINK OF IT AS " $\dfrac{\text{\% CHANGE } q}{\text{\% CHANGE } p}$ "

• SAW THAT $|\varepsilon(p)|$ DETERMINES SIGN OF $R'(p)$ & USED TO SOLVE PROBLEMS

TODAY

COMPOUND INTEREST

SUPPOSE YOU INVEST \$1 INTO AN ACCOUNT WITH AN INTEREST RATE OF 100% PER YEAR, COMPOUNDED n TIMES PER YEAR (ie ADD $\frac{100}{n}$ % OF CURRENT VALUE TO ACCOUNT n TIMES PER YEAR)

WHAT IS THE VALUE OF YOUR ACCOUNT AFTER ONE YEAR IF YOU COMPOUND INTEREST (II)

$n=1$ TIMES $\boxed{?}$

$\frac{100\%}{1} = 1$

$1 + 1 \cdot 1 = \$2$

$n=2$ TIMES $\boxed{?}$

$\frac{100\%}{2} = \frac{1}{2}$

$$1 + \left(\frac{1}{2}\right)\cdot 1 + \left(\frac{1}{2}\right)\left(1 + \left(\frac{1}{2}\right)\cdot 1\right)$$

$$= \left[1 + \frac{1}{2}\right] + \left(\frac{1}{2}\right)\left[1 + \frac{1}{2}\right]$$

$$= \left[1 + \frac{1}{2}\right]\left(1 + \frac{1}{2}\right)$$

$$= \left(1 + \frac{1}{2}\right)^2 = \$2.25$$

$n=3$ TIMES $\boxed{?}$

$$\underbrace{1 + \frac{1}{3} + \left(\frac{1}{3}\right)\left[1 + \frac{1}{3}\right]} + \left(\frac{1}{3}\right)\left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)\left[1 + \frac{1}{3}\right]\right]$$

$$= \left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{3}\right) + \left(\frac{1}{3}\right)\left[\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{3}\right)\right]$$

$$= \left(1 + 1/3\right)\left(1 + 1/3\right)\left(1 + 1/3\right) = \left(1 + 1/3\right)^3 = \$2.37$$

HIGHER COMPOUNDING FREQUENCY LEADS TO HIGHER INVESTMENT VALUE AFTER ONE YEAR! (III)

n = 365 TIMES?

$$\left(1+\frac{1}{365}\right)^{365} = \$2.71$$

n TIMES?

$$\$\left(1+\frac{1}{n}\right)^{n}$$

WHAT HAPPENS IF WE LET $n \to \infty$ $\boxed{?}$

(THIS IS CALLED CONTINUOUS COMPOUNDING)

RETURN AFTER ONE YEAR IS

$$\boxed{\$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}}$$

$\boxed{Q}$ CAN WE EVALUATE THIS LIMIT?

WHAT IS YOUR GO-TO METHOD TO HANDLE ANNOYING POWERS?

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \lim_{n\to\infty} e^{\ln\left[\left(1+\frac{1}{n}\right)^n\right]}$$ IV

$$= e^{\underbrace{\lim_{n\to\infty}\ln\left[\left(1+\frac{1}{n}\right)^n\right]}_{\text{FOCUS ON THIS}}}$$

FOR CTS FUNCTIONS
LIMIT OF COMPOSITE
IS COMPOSITE OF LIMIT

$$\lim_{n\to\infty}\ln\left[\left(1+\frac{1}{n}\right)^n\right] = \lim_{n\to\infty} n\ln\left(1+\frac{1}{n}\right)$$

FUNNY STEP

$$= \lim_{n\to\infty}\frac{\ln\left(1+\frac{1}{n}\right)}{\left(1/n\right)}$$

LET $h = \frac{1}{n}$

THIS MEANS $\boxed{?}$ $h \to 0$

$$= \lim_{h\to 0}\frac{\ln(1+h)}{h}$$

CAN YOU SPOT A DERIVATIVE ?

$$= \lim_{h\to 0}\frac{\ln(1+h)-\ln(1)}{h}$$

$$= \left(\frac{d}{dx}\left[\ln(x)\right] \text{ AT } x=1\right) = \boxed{1}$$

$$\Rightarrow \boxed{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e}$$

V

ie GET \$e AFTER ONE YEAR!
OR \$$e^{t}$ AFTER t YEARS

IN GENERAL IF YOU:

- INVEST \$$A_0$
- WITH AN INTEREST RATE OF r% / YEAR
- COMPOUND n TIMES PER YEAR

THEN AFTER $t \in \mathbb{N}$ YEARS YOU HAVE

$$\boxed{\$A(t) = \$A_0\left(1+\frac{r}{n}\right)^{nt}}$$

IF, INSTEAD, YOU COMPOUND CONTINUOUSLY, AFTER $t \in \mathbb{N}$ YEARS YOU HAVE

$$\boxed{\$A(t) = \$\lim_{n\to\infty} A_0\left(1+\frac{r}{n}\right)^{nt} = \$A_0 e^{rt}}$$

$\boxed{\text{HW}}$ SEE HOW YOU WOULD OBTAIN THESE FORMULAS

VI

EXAMPLE

INVEST \$100 WITH INTEREST RATE OF 5% PER YEAR. WHAT IS THE BALANCE IN YOUR ACCOUNT AFTER 3 YEARS IF INTEREST IS COMPOUNDED

ⓐ YEARLY

$t=3$, $n=1$, $r=0.05$, $A_0=\$100$

$$A(3) = \$100\left(1+\frac{0.05}{1}\right)^{1\cdot 3} = \$115.76$$

ⓑ QUARTERLY

$$A(3) = \$100\left(1+\frac{0.05}{4}\right)^{4\cdot 3} = \$116.08$$

ⓒ CONTINUOUSLY

$$A(3) = \$100\, e^{(0.05)(3)} = \$116.18$$

⟹ NOTES + PROBLEMS POSTED ON COURSE WEB SITE

THIS IS AN EXAMPLE OF

VII

EXPONENTIAL GROWTH

USUALLY EXPONENTIAL FUNCTIONS OCCUR AS SOLUTIONS TO THE FOLLOWING KIND OF PROBLEM:

SUPPOSE WE MODEL A POPULATION BY OBSERVING THAT:

"THE RATE OF CHANGE IN THE POPULATION IS PROPORTIONAL TO ITS SIZE"

ie. Q

$$\boxed{\frac{d}{dt}[P(t)] = k \cdot P(t)}$$

PROPORTIONALITY CONSTANT

WHAT CAN $P(t)$ BE?

(A FUNCTION WHOSE DERIVATIVE IS VERY CLOSE TO ORIGINAL FUNCTION)

$P(t) = Ce^{kt}$ WORKS

$$P'(t) = kCe^{kt} = kP(t) \checkmark$$

WHAT SHOULD C BE?

$$P(0) = Ce^{k0} = C \cdot 1 = C$$

$$\Rightarrow \boxed{P(t) = P(0) \cdot e^{kt}}$$

EX) IF $k > 0$, HOW LONG DOES IT TAKE FOR POPULATION TO DOUBLE?

NEED TO SOLVE FOR t $\boxed{2P(0) = P(0) \cdot e^{kt}}$

$$\Rightarrow 2 = e^{kt} \Rightarrow \ln(2) = kt$$

$$\Rightarrow \boxed{t = \frac{\ln(2)}{k}}$$

DOESN'T DEPEND ON $P(0)$ AT ALL!

$\hookrightarrow$ EXTRA NOTES & PROBLEMS POSTED ONLINE TO HELP WITH WEBWORK

HAVE A NICE WEEKEND.