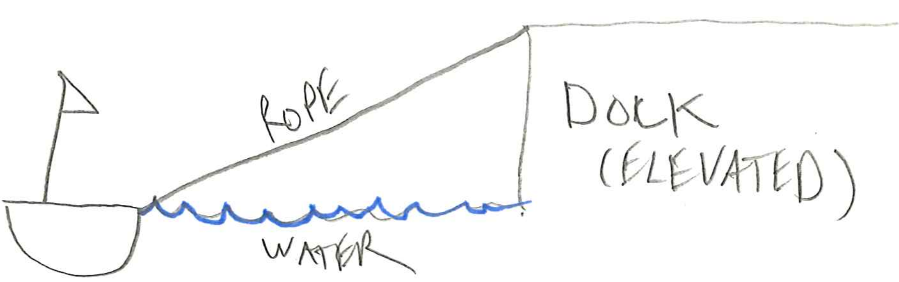
TODAY WE END OUR EXCURSION INTO THE WORLD OF APPLICATIONS BY FOCUSING ON THE INTERPRETATION OF DERIVATIVES AS RATES OF CHANGE IN PHYSICAL PROBLEMS (SO-CALLED RELATED RATES PROBLEMS). WE WILL RETURN TO THE MORE GRANDIOSE TASK OF UNDERSTANDING THE SHAPE OF GRAPHS ON WEDNESDAY.

FOR THE FOLLOWING PROBLEMS

1. USE YOUR INTUITION TO TRY & GUESS THE ANSWER
2. TALK ABOUT YOUR GUESS WITH THE PERSON NEXT TO YOU
3. USE MATHEMATICS TO VERIFY IF YOUR INTUITION WAS CORRECT



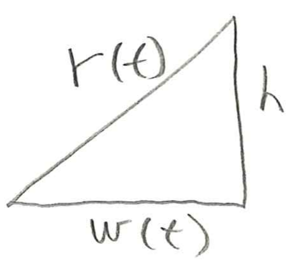
[Q] A BOAT IS BEING DRAWN CLOSE TO A DOCK BY PULLING A ROPE AS SHOWN. II

HOW IS THE RATE AT WHICH THE ROPE IS BEING PULLED RELATED TO THE RATE AT WHICH THE BOAT APPROACHES THE DOCK?

(A) ONE IS A CONSTANT MULTIPLE OF THE OTHER

(B) THEY ARE EQUAL

(C) IT DEPENDS ON HOW CLOSE THE BOAT IS TO THE DOCK.



$$(r(t))^2 = h^2 + (w(t))^2$$

$$2\,r(t)\,\frac{d}{dt}(r(t)) = 2\,w(t)\,\frac{d}{dt}(w(t))$$

$$\Rightarrow r(t)\,\frac{d}{dt}(r(t)) = w(t)\,\frac{d}{dt}w(t)$$

IT DEPENDS

[Q] SUPPOSE THE BOAT IS BEING DRAWN CLOSE TO THE DOCK BY PULLING THE ROPE AT A CONSTANT RATE.

III

[T/F] THE CLOSER THE BOAT GETS TO THE DOCK, THE FASTER IT IS MOVING.

WELL $\boxed{r\dfrac{dr}{dt} = w\dfrac{dw}{dt}}$

$$\Rightarrow \boxed{\frac{r}{w}\underbrace{\frac{dr}{dt}}_{\text{CONSTANT}} = \frac{dw}{dt}}$$

$r^2 = w^2 + h^2$

$\Rightarrow r = \sqrt{w^2+h^2}$

$$\Rightarrow \boxed{\frac{\sqrt{w^2+h^2}}{w}\frac{dr}{dt} = \frac{dw}{dt}}$$

AS $w \to 0$ LHS GOES TO $+\infty$

WTF!? WHY IS OUR MODEL WRONG?

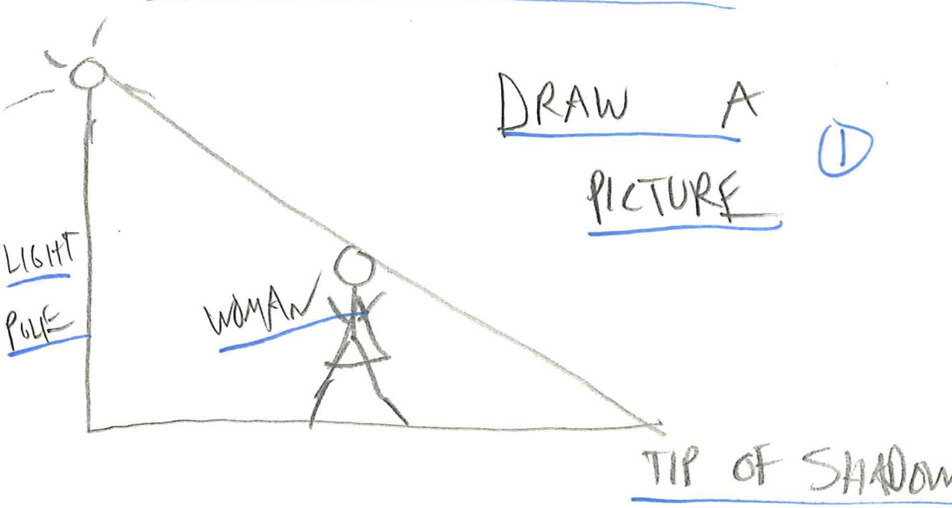
[Q] A STREET LIGHT IS MOUNTED AT THE TOP OF A POLE. A WOMAN WALKS AWAY FROM THE POLE. HOW ARE THE RATE AT WHICH SHE WALKS AWAY FROM THE POLE & THE RATE AT WHICH HER SHADOW GROWS RELATED?

IV

(A) ONE IS A CONSTANT MULTIPLE OF THE OTHER

(B) THEY ARE EQUAL

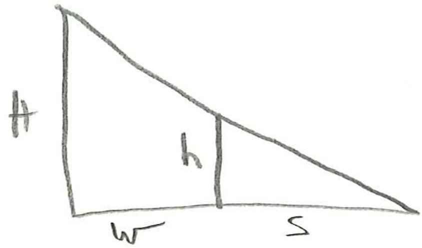
(C) IT DEPENDS ON HOW CLOSE SHE IS TO THE POLE



DRAW A PICTURE

(1)

V



LABEL RELEVANT VALUES ②

WHICH ONES ARE CONSTANTS?

WHICH ONES ARE VARIABLES?

CAN WE FIND AN EQUATION RELATING THE FUNCTIONS?

$$\frac{s}{h} = \frac{w+s}{H}$$

SIMILAR TRIANGLES

$$\left(\frac{1}{h}\right)\cdot s(t) = \left(\frac{1}{H}\right)\left(w(t)+s(t)\right)$$

$$\left(\frac{1}{h}\right)\frac{ds}{dt} = \left(\frac{1}{H}\right)\left(\frac{dw}{dt}+\frac{ds}{dt}\right)$$

$$\left(\frac{1}{h}\right)\frac{ds}{dt} - \left(\frac{1}{H}\right)\frac{ds}{dt} = \left(\frac{1}{H}\right)\frac{dw}{dT}$$

$$\left(\frac{1}{h}-\frac{1}{H}\right)\frac{ds}{dt} = \left(\frac{1}{H}\right)\frac{dw}{dT}$$

A

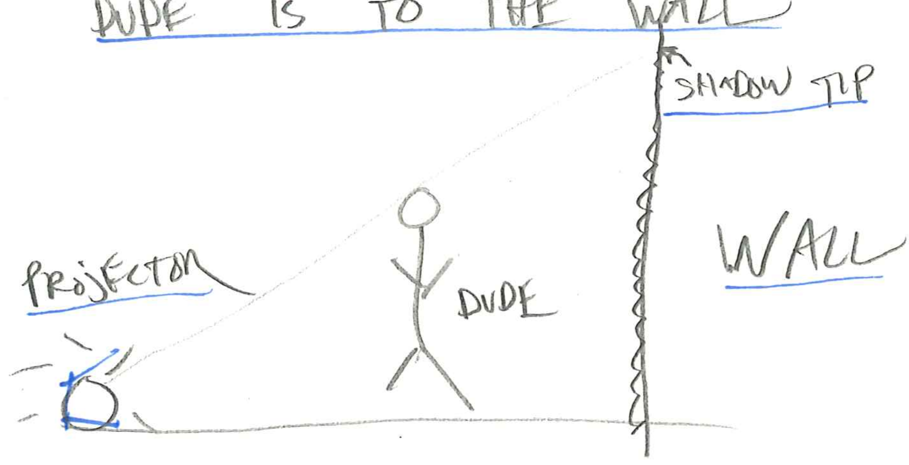
[Q] A SPOTLIGHT INSTALLED ON THE GROUND SHINES ON A WALL. SOME DUDE STANDS BETWEEN THE LIGHT AND THE WALL, CASTING A SHADOW ON THE WALL. HOW ARE THE RATE AT WHICH HE WALKS AWAY FROM THE LIGHT & THE RATE AT WHICH HIS SHADOW GROWS RELATED?

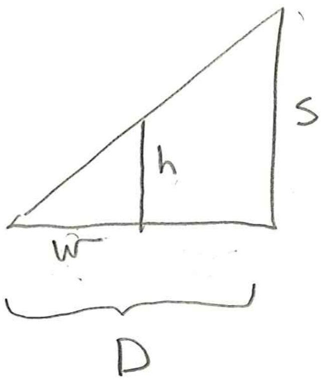
VI

(A) ONE IS A CONSTANT MULTIPLE OF OTHER

(B) THEY ARE EQUAL

(C) IT DEPENDS ON HOW CLOSE THE DUDE IS TO THE WALL





VII

FIND AN EQUATION RELATING THE QUANTITIES OF INTEREST

$$\boxed{\frac{S}{D} = \frac{h}{w}} \Rightarrow \left(\frac{1}{D}\right) \cdot S(t) = h \cdot \frac{1}{w(t)}$$

TAKE $\frac{d}{dt}$ $\left(\frac{1}{D}\right) \cdot \frac{dS}{dt} = (h) \cdot \frac{-1}{w^2} \cdot \frac{dw}{dt}$

DEPENDS ON w

AKA DEPENDS ON $D-w$ = DISTANCE FROM WALL

WHAT IF WE DID IT DIFFERENTLY

$$\boxed{S \cdot w = h \cdot D}$$

TAKE $\frac{d}{dt}$ ON BOTH SIDES

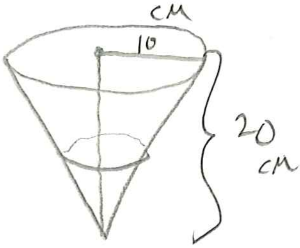
$$\left(\frac{dS}{dt}\right) w + S\left(\frac{dw}{dt}\right) = 0$$

$$\Rightarrow \boxed{\frac{dS}{dt} = -\frac{S}{w} \frac{dw}{dT}}$$

IS THIS THE SAME ANSWER WE HAD BEFORE ! ? !

CAUTION: SIMILAR SETUP; DIFFERENT OUTCOMES!

BEER FLOWS OUT BOTTOM OF FUNNEL (VIII)

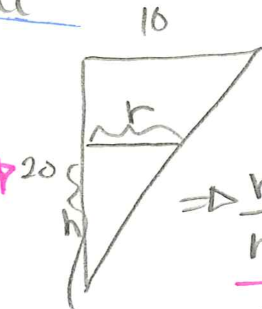


YOU MEASURE VOLUME & DEPTH OF BEER IN FUNNEL AS IT FLOWS, V & h

(a) EXPRESS V AS FUNCTION OF h & USE THIS TO EXPRESS $\frac{dV}{dt}$ IN TERMS OF h & $\frac{dh}{dt}$

$$\boxed{V = \left(\frac{1}{3}\right)\pi r^2 h}$$

$$\boxed{V = \frac{\pi h^3}{12}}$$



$\Rightarrow \frac{r}{h} = \frac{10}{20}$

$\Rightarrow r = \frac{h}{2}$

$$\Rightarrow \boxed{\frac{dV}{dt} = \frac{\pi}{4} h^2 \cdot \frac{dh}{dt}}$$

(b) $\frac{dV}{dt} = c\sqrt{h}$

SIGN (C)?

UNITS OF (C)?

- C NEGATIVE B/C V DECREASING
- $\left(\text{cm}^3/\text{s}\right) \cdot \frac{1}{\sqrt{\text{cm}}} = (\text{cm})^{5/2}/\text{s}$

(c) SHOW $\frac{dh}{dt} = \frac{4c}{\pi} h^{-3/2}$ (IX)

COMBINE (a) & (b)

$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt} = c\sqrt{h}$$

& JUST SOLVE FOR

(d) IF BEER LEVEL FALLS AT 5cm/s WHEN $h = 4$cm WHAT IS c?

FROM (c) $\frac{dh}{dt} = \frac{4}{\pi} c h^{-3/2}$

$$\Rightarrow (-5) = \frac{4}{\pi} c \, (4)^{-3/2}$$

$$\Rightarrow (-5) = \left(\frac{4}{\pi}\right) \cdot c \cdot \left(\sqrt{4}\right)^{-3}$$

$$\boxed{-10\pi = c}$$

cm$^{5/2}$/s

READ 3.11

(WE SKIPPED 3.10)