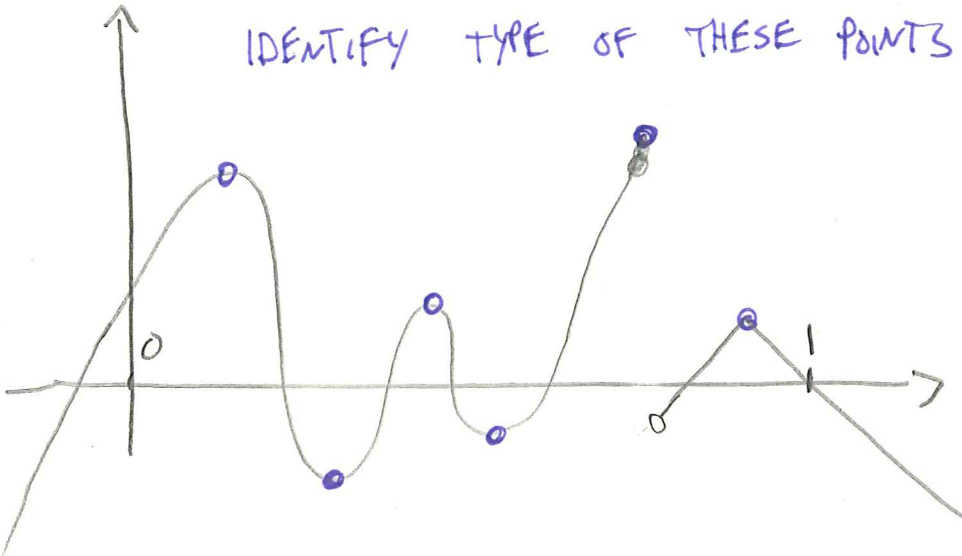
LAST TIME

ABSOLUTE VS LOCAL MAXIMUM & MINIMUM



POWERFUL THM (ANOTHER WAY CONTINUITY AFFECTS THE SHAPE OF GRAPHS)

EXTREME VALUE THM

IF f IS CTS ON $[a, b]$

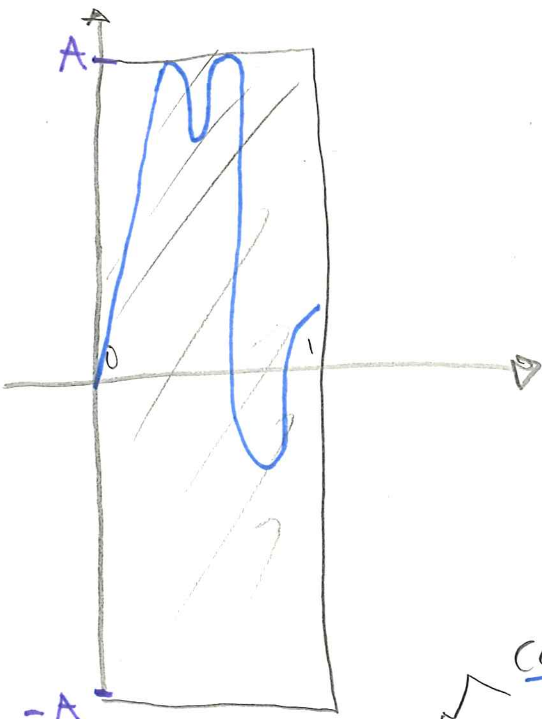
THEN f HAS (AT LEAST) A GLOBAL MAX $f(c)$ & (AT LEAST) A GLOBAL MIN $f(d)$, $a \leq c \leq b$ & $a \leq d \leq c$

Q: WHAT ABOUT THE FUNCTION I DREW ON $[0, 1]$?

A LITMUS TEST

II

LET f BE CTS ON $[0,1]$.
THEN (~~THE EVT TELLS US~~) THERE IS A POSITIVE CONSTANT A SO THAT THE GRAPH OF f IS CONTAINED WITHIN THE RECTANGLE $0 \le x \le 1$
$-A \le y \le A$.



THE ABOVE STATEMENT IS

ⓐ ALWAYS TRUE

ⓑ SOMETIMES TRUE

ⓒ NOT ENOUGH INFO TO TELL

CONSTANTS

EVT SAYS $\exists$ $m \le f(x) \le M$

SET A = MAXIMUM OF m & M

VOILÀ YOU'VE PUT YOUR FUNCTION IN A BOX

III

HW THINK OF A CTS FUNCTION ON (0,1) WHICH YOU CAN'T PUT IN A BOX

OUR GRAND SCHEME SO FAR

REAL WORLD PROBLEM → COMPLICATED RELATIONSHIPS ARE EASILY EXPRESSED IN LANGUAGE OF FUNCTIONS → OFTEN WANT TO FIND MAX & MINS

USUALLY, FIND CTS FUNCTIONS, SO WE MADE PROGRESS, WE KNOW THE MAX/MIN WE SEEK EXISTS (NOT OBVIOUS)

BUT ONE Q REMAINS UNADDRESSED.

WHERE ARE THEY !?

DON'T USUALLY HAVE EASY ACCESS TO A GRAPH TO "LOOK" FOR THEM (EVEN WOLFRAM CAN'T DO IT SOMETIMES)

NEED TO BE CLEVER

ANSWER LIES IN DERIVATIVES!

TO SEE HOW THIS WORKS IV

SUPPOSE f IS DIFF AT a

$\Rightarrow$ THREE CASES

(i) $f'(a) > 0$

CASE OF INTEREST $\rightarrow$ $\boxed{\text{(ii) } f'(a) = 0}$

(iii) $f'(a) < 0$

TELL ME

WHAT DO THEY MEAN GEOMETRICALLY?

ON OTHER HAND IF f IS NOT DIFF, WE HAVE NO IDEA WHAT HAPPENS

(FORTUNATELY MOST FUNCTIONS WE ENCOUNTER ARE NOT DIFF ONLY AT A FEW PLACES)

NICE CRITERION

LOCAL EXTREME VALUE THM

IF f HAS A LOCAL MAX OR MIN AT c THEN $f'(c) = 0$ $\boxed{\text{OR}}$ $f'(c)$ DNE

SINCE THESE PTS ARE CRITICAL IN OUR SEARCH WE CALL THEM CRITICAL POINTS

DOES THIS SAY CRITS ARE MAXIMA & MINIMA ?

EX $\bullet$ $f(x) = x^2 \rightsquigarrow f'(x) = 2x$ (V)

ONLY ONE CRIT POINT & IT IS A GLOBAL MIN

$\bullet$ $f(x) = x^3 \rightsquigarrow f'(x) = 3x^2$ DRAW

ONLY ONE CRIT POINT BUT IT IS NOT A MAX/MIN

$\bullet$ $f(x) = |x|$ HAS GLOBAL MIN AT 0

WHICH IS A CRIT POINT B/C $f'(0)$ DNE

OK BACK TO A CTS FUNCTION ON A

CLOSED INTERVAL $[a, b]$

WHERE CAN ITS ABSOLUTE MAX & MINS

BE HIDING? I WANT AN ANSWER

CLOSED INTERVAL METHOD

TO FIND THE GLOBAL MAX/MIN OF

A CTS FUNCTION f ON $[a, b]$

1. FIND ALL CRIT POINTS IN (a, b) & COMPUTE f AT THESE POINTS
2. FIND VALUE OF f AT END POINTS a & b
3. LARGEST OF THE ABOVE IS GLOBAL MAX

   SMALLEST IS GLOBAL MIN

EX] FIND GLOBAL MAX/MIN OF VI

$\boxed{f(x) = x^{5/3} - x^{2/3}}$ ON $[-1, 1]$

$f'(x) = \frac{5}{3}x^{2/3} - \frac{2}{3}x^{-1/3}$

$= x^{-1/3}\left(\frac{5}{3}x - \frac{2}{3}\right)$

$f'(x) = 0$ WHEN $x = 2/5$ & $f'(x)$ DNE WHEN $x = 0$

$\left.\begin{array}{l} f(0) = 0 \\ f(2/5) = \left(\frac{2}{5}\right)^{2/3}\left(2/5 - 1\right) \end{array}\right\}$ CRITS

$\left.\begin{array}{l} f(-1) = (-1)^{2/3}(-2) = -2 \\ f(1) = 1 \cdot 0 = 0 \end{array}\right\}$ ENDPOINTS

$\Rightarrow$ ABS MAX $= 0$.

ABS MIN $= -2$

TRY & SKETCH THE GRAPH INSTEAD

WHEN THE FUNCTION WE WISH

TO UNDERSTAND DIFF THIS MACHINERY

YIELDS MORE POWERFUL TECHNIQUES

& A GENERAL UNDERSTANDING

OF THE SHAPE OF GRAPHS

↳ THIS WILL BE DONE ON

MONDAY BY A GUEST LECTURER

" $\boxed{\text{ED}}$ "

HAVE A NICE WEEK END!

[redacted]