MIDTERM WEDNESDAY NOVEMBER 12 @6:30 PM (I)

↳ HAVE A CONFLICT? E-MAIL ME $\boxed{\text{TODAY}}$

OFFICE HOURS TOMORROW 2→3 PM

THIS WEEK IS ALL ABOUT JUSTIFYING THE $\boxed{\text{TIME}}$ & $\boxed{\text{EFFORT}}$ SPENT LEARNING HOW TO COMPUTE DERIVATIVES & SEEING HOW FOR DIFF FUNCTIONS KNOWING A $\boxed{\text{LITTLE}}$ ABOUT $f'(x)$ TELLS US A $\boxed{\text{LOT}}$ ABOUT $f(x)$

[Q1] LET f BE DIFF & CONSIDER ITS RESTRICTION TO $[a,b]$.

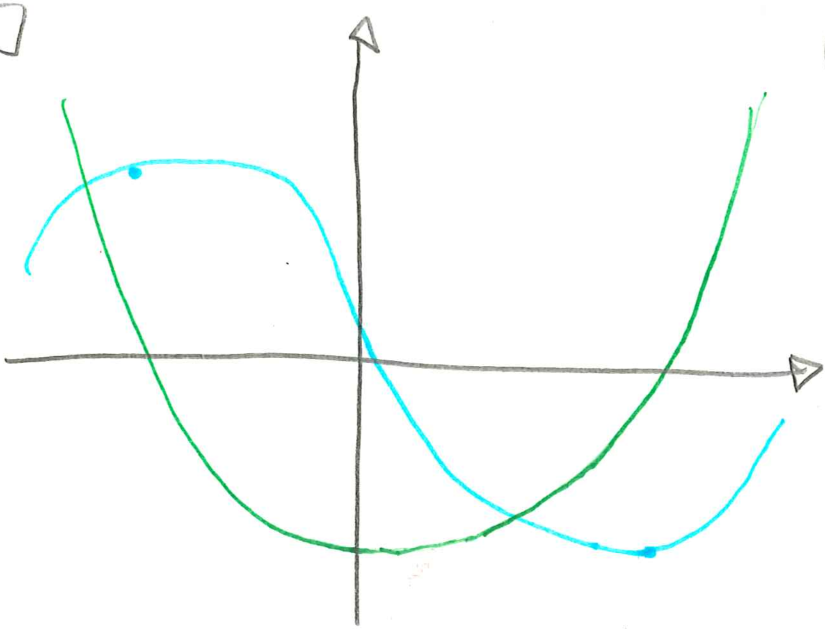
IF $f'(a)>0$ THEN

1. f COULD HAVE AN ABSOLUTE MAX OR AN ABSOLUTE MIN AT a
2. f CANNOT HAVE AN ABSOLUTE MAX AT a
3. f CANNOT HAVE AN ABSOLUTE MIN AT a.

↳ DRAW SOME EXAMPLES!

[Q2]

II



WHO IS f? WHO IS f'? WHY?

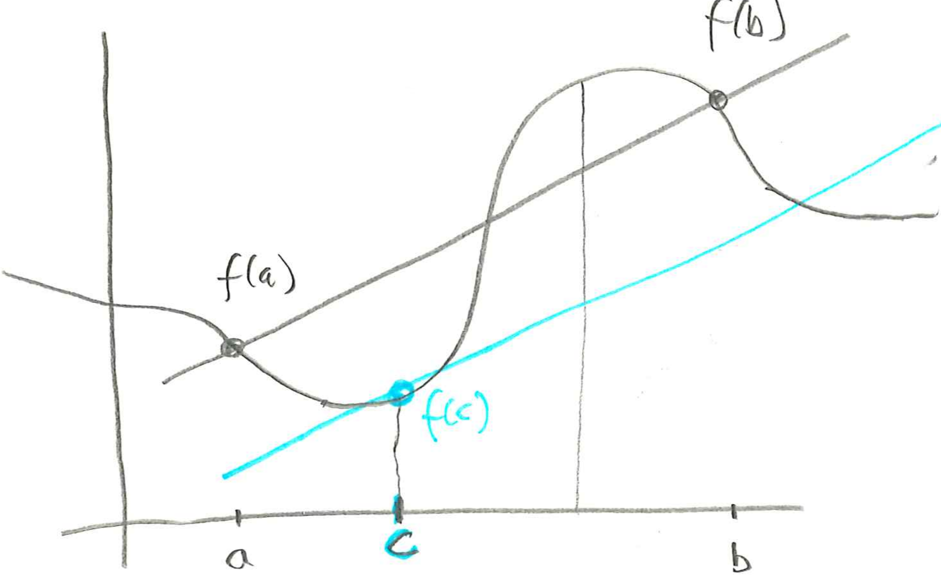
[Q3] IF $f' = 0$ MUST f BE CONSTANT?

LAST TIME GOT A LITTLE LOST IN INCEPTION SO LET'S BE MORE PRECISE BY INTRODUCING THE MOST POWERFUL TOOL OF DIFF CALC

A SOUPPED-UP VERSION OF GEOMETRIC INTERPRETATION OF $f'(c)$ AS SLOPE OF TANGENT LINE TO CURVE $y = f(x)$ AT POINT c

KEY TO EVERYTHING e.g. IF $f' = 0$ MUST f BEC

MEAN VALUE THM



IF f IS DIFF ON (a,b) &

CTS ON $[a,b]$ THEN THERE IS

SOME $a < c < b$ FOR WHICH

$$\boxed{\frac{f(b)-f(a)}{b-a} = f'(c)}$$

CHALLENGE: • USE ^{EASY} EXTREME VALUE THM TO PROVE MVT WHEN $f(a)=f(b)=0$

• USE ^{HARD} THIS SPECIAL CASE TO PROVE FULL MVT.

THE MVT IS THE PRECISE MATH BEHIND THE HEURISTIC

"IF $f' > 0$ ON AN INTERVAL THEN f IS INCREASING ON THE INTERVAL"

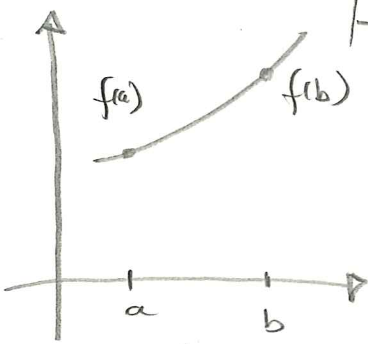
HOW DO WE SHOW THIS

WHAT DO WE KNOW? $f'(x) > 0$

WHAT IS OUR GOAL? TO SHOW THAT

$\boxed{f \text{ IS INCREASING}}$

i.e. IN MATH: $\boxed{\text{IF } a < b \text{ THEN } f(a) < f(b)}$



WHAT DOES MVT TELL US?

FOR SOME $a < c < b$

WE HAVE $\dfrac{f(b) - f(a)}{b - a} = f'(c)$

WHAT DO WE KNOW ABOUT $f'(c)$ HERE? $\boxed{>0 \,!!!}$

$$\frac{f(b)-f(a)}{b-a} = f'(c) > 0$$

$$\Rightarrow \frac{f(b)-f(a)}{b-a} > 0$$

$$\Rightarrow f(b)-f(a) > 0$$ (B/C WHAT?)

$$\Rightarrow f(b) > f(a)$$ TADA! ■

SIMILARILY CAN SHOW (DO IT)

"IF $f' < 0$ ON AN INTERVAL THEN f IS DECREASING ON IT"

THIS IS CALLED THE INCREASING / DECREASING TEST!

TURNS INFO ABOUT f' INTO INFO ABOUT SHAPE OF GRAPH OF f

HOW DOES THIS WORK IN PRACTISE? VI

$$\boxed{f(x) = x^3 - x}$$

WHAT DOES IT LOOK LIKE?

USE $f'(x)$ TO UNDERSTAND

$$f'(x) = 3x^2 - 1$$

CRITICAL POINTS $\boxed{3x^2 - 1 \Rightarrow x = \pm\sqrt{1/3}}$

$\hookrightarrow$ GET SHAPE INFO EVERYWHERE ELSE

WHERE IS $f'(x) > 0$?

$$\left.\begin{aligned} &3x^2 - 1 > 0 \\ \Rightarrow\ &3x^2 > 1 \\ \Rightarrow\ &x^2 > 1/3 \end{aligned}\right\} \quad x > \sqrt{1/3} \text{ OR } x < -\sqrt{1/3}$$

$f'(x) < 0$?

$$\left.\begin{aligned} &3x^2 - 1 < 0 \\ \Rightarrow\ &x^2 < 1/3 \end{aligned}\right\} \quad -\sqrt{1/3} < x < \sqrt{1/3}$$

SUMMARIZE

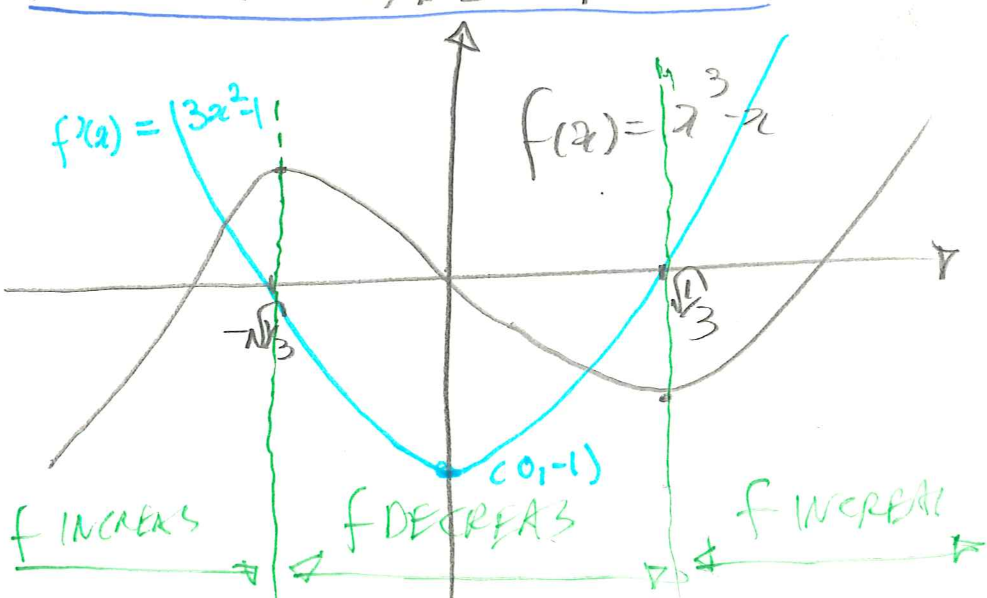
INTERVAL	$f'(x)$	$f(x)$
$x < -\sqrt{1/3}$	>0	INCREAS
$-\sqrt{1/3} < x < \sqrt{1/3}$	<0	DECREAS
$x > \sqrt{1/3}$	>0	INCREAS

NEED TO KNOW

$$f\left(\sqrt{1/3}\right) = -\frac{2}{3}\sqrt{1/3}$$

$$f\left(-\sqrt{1/3}\right) = \frac{2}{3}\sqrt{1/3}$$

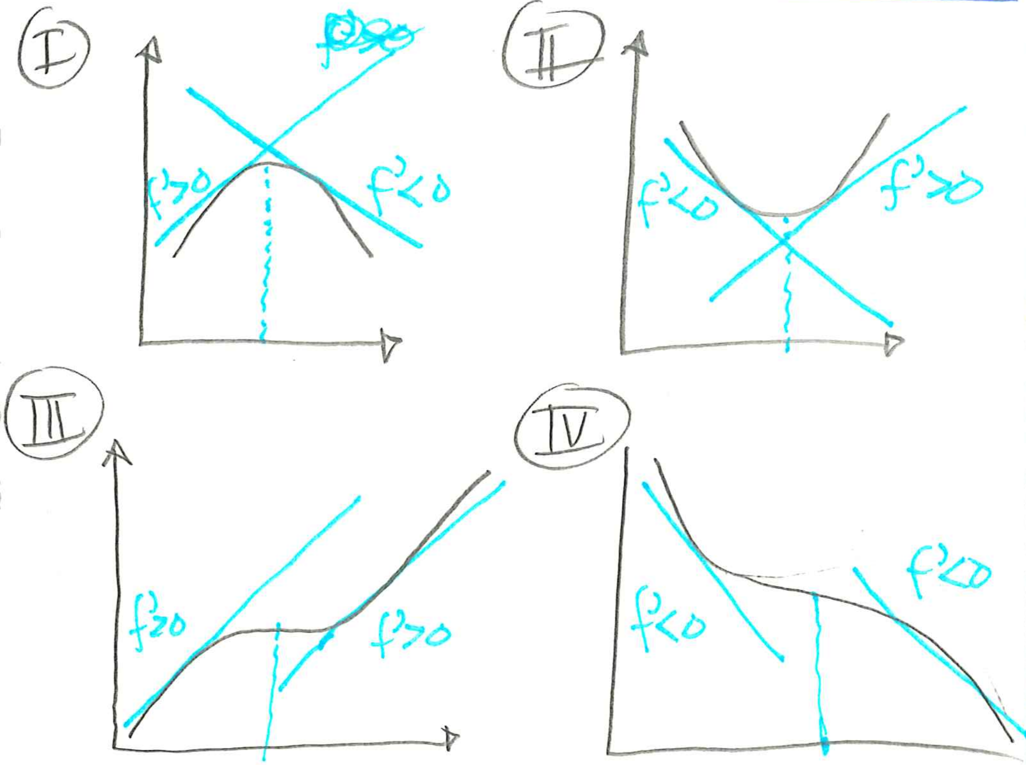
& GET REASONABLE PLOT



VIIA

DIDN'T NEED TO SKETCH TO LOCATE LOCAL MAX / MIN

JUST ANALYZE SIGN CHANGES OF f'



FIRST DERIVATIVE TEST

SUPPOSE c IS A CRIT POINT OF f

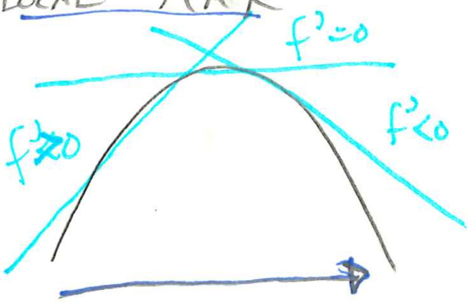
1. f' CHANGES FROM POSITIVE TO NEGATIVE AT $c \Rightarrow f$ HAS LOCAL MAX
2. f' CHANGES FROM NEGATIVE TO POS AT $c \Rightarrow f$ HAS LOCAL MIN
3. OTHERWISE, NO LOCAL EXTREMUM

KNOWING A LITTLE (ITS SIGN) ABOUT f' TOLD US A LOT (INCREASING / DECREASING) ABOUT f

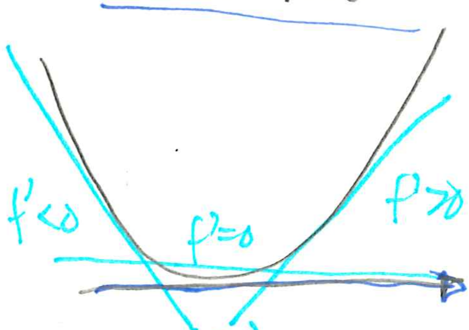
WHAT IF WE HAD MORE KNOWLEDGE ABOUT f'?

CAN WE CHARACTERIZE SITUATIONS WE CARE ABOUT IN ANOTHER WAY?

LOCAL MAX



LOCAL MIN



NOTICE f' DECREASES

NOTICE f' INCREASES

CONCAVE DOWN

CONCAVE UP

OBSERVATIONS WILL LEAD US TO SECOND DERIVATIVE TEST!