MIDTERM COMING UP: NOVEMBER 12 (I)

6:30 → 7:30

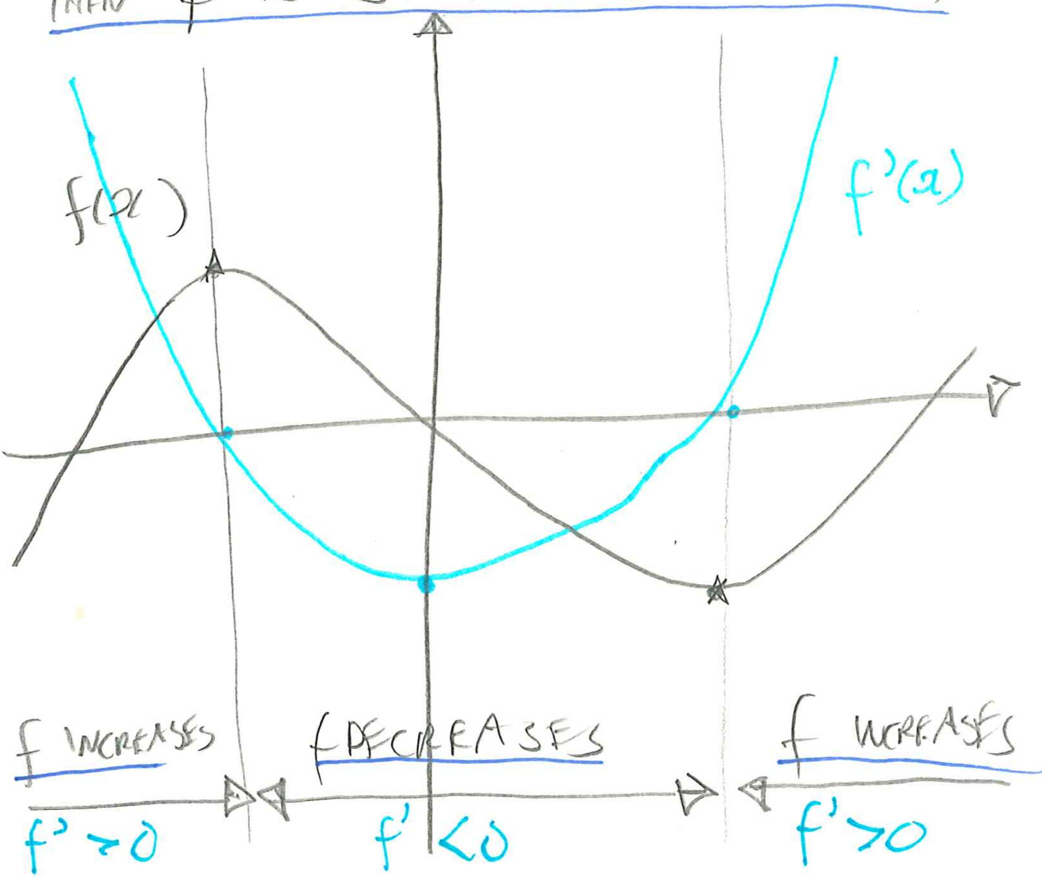
GIVE THE PRACTISE MIDTERM A SHOT THIS WEEK END! (ONCE YOU'VE RECOVERED FROM HALLOWEEN)

LAST TIME

INCREASING / DECREASING TEST

IF $f' > 0$ ON AN INTERVAL (a, b)
THEN f IS INCREASING ON (a, b)

IF $f' < 0$ ON AN INTERVAL (a, b)
THEN f IS DECREASING ON (a, b)



FIRST DERIVATIVE TEST SAYS (II)

LOCAL MAX / MIN HAPPEN WHEN f' CHANGES SIGN ~~(IF f' EXISTS)~~

GRAB SOME CHOCOLATE

[Q] CONSIDER $\boxed{f(x) = \frac{x^2-9}{x^2+3}}$

(1) FIND ALL LOCAL MAX / MINS

(2) SKETCH THE GRAPH

(1) $$f'(x) = \frac{(2x)(x^2+3) - (x^2-9)(2x)}{(x^2+3)^2}$$

EXISTS EVERYWHERE, NEVER ZERO

CRIT POINTS

$$0 = \frac{(2x)(x^2+3) - (x^2-9)(2x)}{(x^2+3)^2}$$

$$\Rightarrow 0 = 2x\left[(\cancel{x^2}+3) - (\cancel{x^2}-9)\right]$$

$$\Rightarrow 0 = 2x[12] = 24x$$

$$\Rightarrow \boxed{0 = x}$$

SINGLE CRITICAL POINT!

III

INTERVAL	f'	f
$(-\infty, 0)$	<0	DECREASING
$(0, +\infty)$	>0	INCREASING

NEED TO FIND SIGN(f')

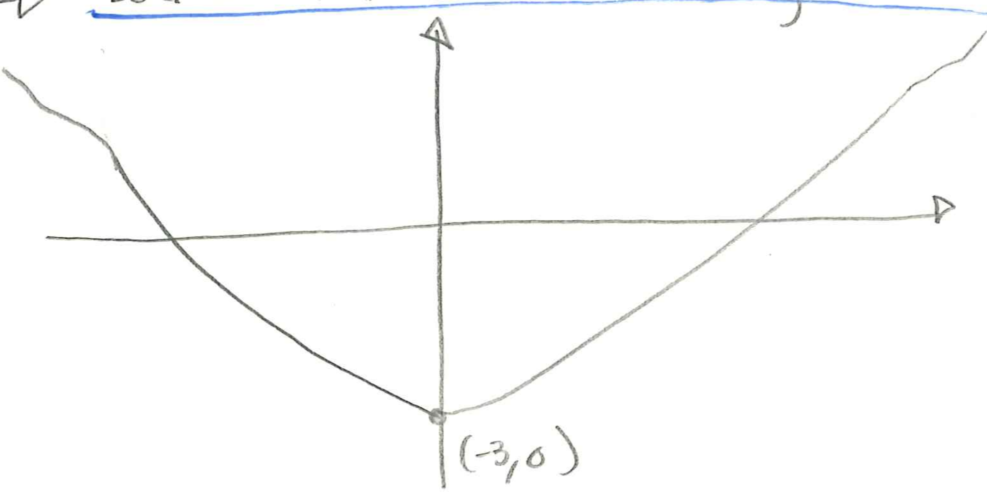
IF (f') IS CTS CAN JUST PLUG

IN A # IN $(-\infty, 0)$

$$f'(-1) = \frac{24(-1)}{((-1)^2+3)^2} < 0$$

$$f'(1) = \frac{24}{(1^2+3)^2} > 0$$

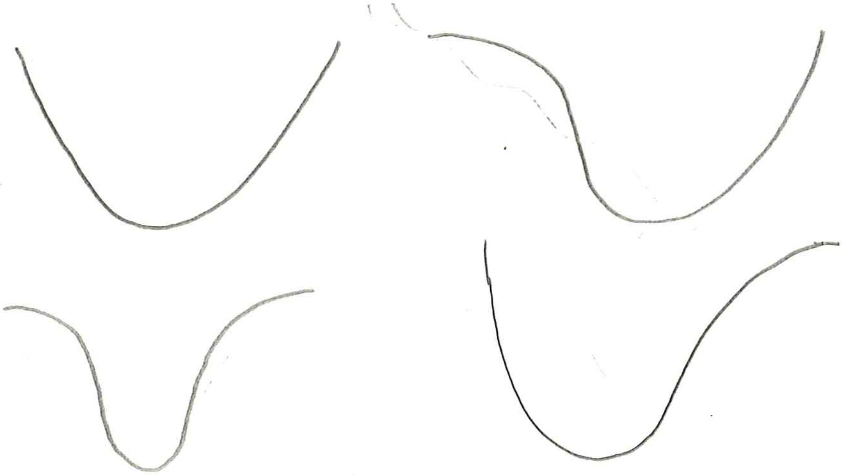
$\Rightarrow$ LOCAL MINIMUM AT 0, $f(0) = \frac{-9}{3} = -3$



HOW ACCURATE IS OUR GRAPH ?

IV

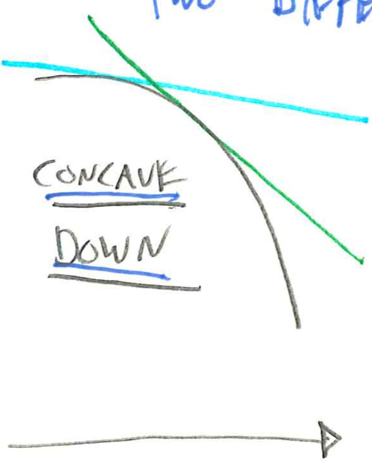
→ ONLY KNOW WHETHER ↑ OR ↓



ALL POSSIBLE

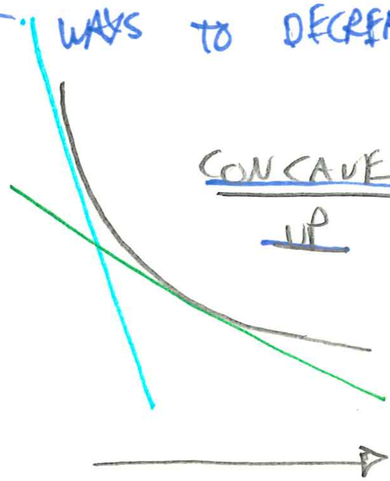
THIS IS WHERE f'' CAN BECOME HELPFUL!

TWO DIFFERENT WAYS TO DECREASE



f' DECREASES

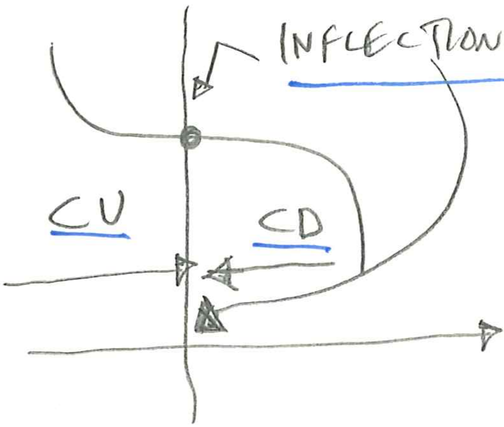
GRAPH TRAPPED BELOW TANGENTS



f' INCREASES

GRAPH TRAPPED ABOVE TANGENTS

FUNNY POINTS WHERE SWITCH BETWEEN TWO CASES ARE CALLED INFLECTION POINTS



INCREASING/DECREASING TEST SAID

$f' > 0 \Rightarrow f$ INCREASING

SO

Q

$f'' > 0 \Rightarrow f'$ INCREASING

INCEPTION

THINK OF f' AS A FUNCTION

$f' < 0 \Rightarrow f$ DECREASING

Q

$f'' < 0 \Rightarrow f'$ DECREASING

INCEPTION

CONCAVITY TEST

$f'' > 0$ ON $(a,b) \Rightarrow f$ CU ON (a,b)

$f'' < 0$ ON $(a,b) \Rightarrow f$ CD ON (a,b)

APPLY THESE IDEAS TO OUR EXAMPLE

VI

$$f'(x) = \frac{24x}{(x^2+3)^2}$$

$$f''(x) = \frac{24(x^2+3)^2 - (24x)(2)(x^2+3)(2x)}{(x^2+3)^4}$$

$$= \frac{[24(x^2+3) - (24x)(2)(2x)](x^2+3)}{(x^2+3)^4}$$

$$= \frac{24x^2+72-96x^2}{(x^2+3)^3}$$

$$= \frac{-72x^2+72}{(x^2+3)^3} = \frac{72(1-x^2)}{(x^2+3)^3}$$

NEED TO FIND CRIT POINTS OF $\boxed{f'}$

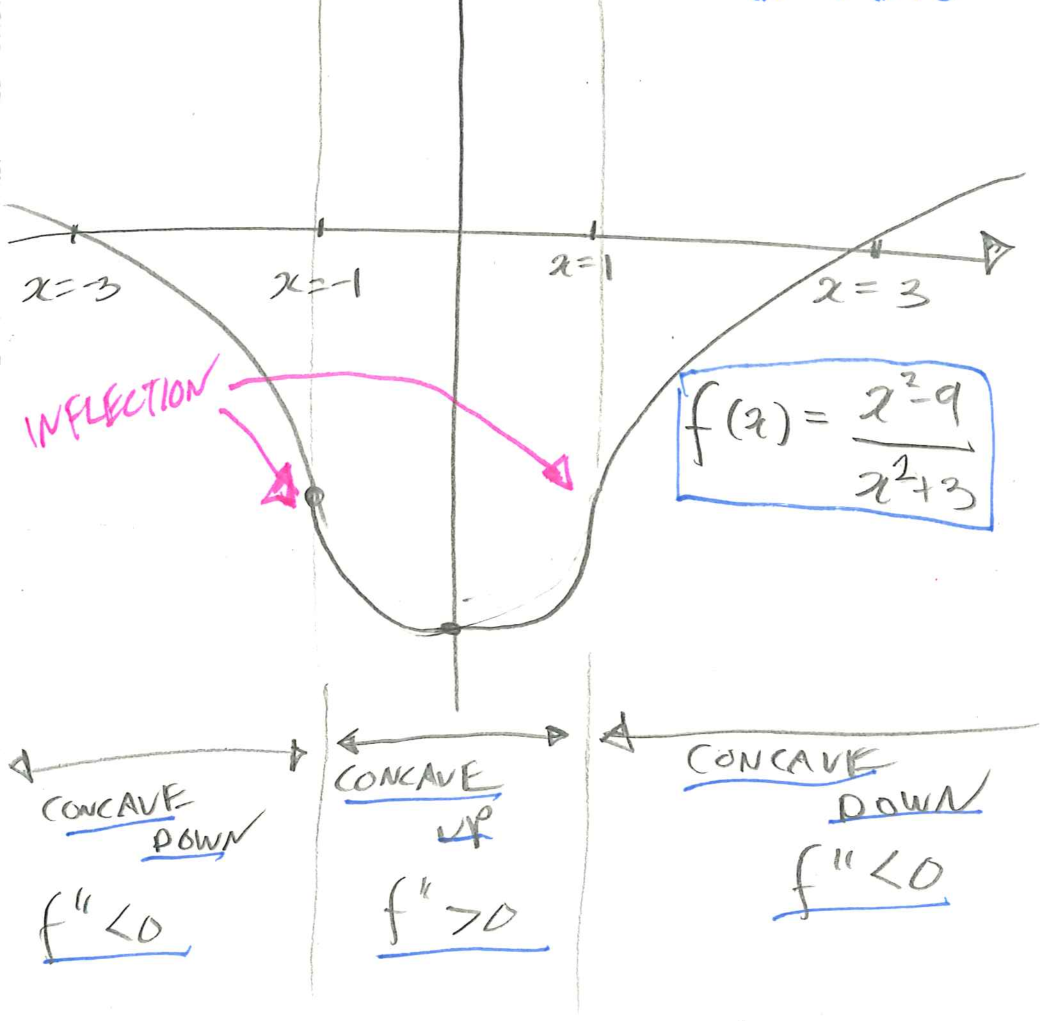
i.e WHERE $f''(x)$ DNE OR $=0$

$$0 = \frac{72(1-x^2)}{(x^2+3)^3} \Rightarrow 0 = (1-x^2)$$

$$\Rightarrow \boxed{x = \pm 1}$$

INTERVAL	f''	CONCAVITY
$(-\infty, -1)$	NEGATIVE	DOWNWARD
$(-1, +1)$	POSITIVE	UPWARD
$(1, +\infty)$	NEGATIVE	DOWNWARD

GET BY PLUGGING IN POINTS



NOTICE: A CRITICAL POINT OF f CONTAINED IN VIII
A CONCAVE UP INTERVAL IS LOCAL
MIN

SECOND DERIVATIVE TEST

IF f'' IS CTS NEAR A CRITICAL
POINT c OF f (ie $f'(c)=0$)

THEN

- $f''(c) < 0 \Rightarrow f(c)$ IS LOCAL MAX
- $f''(c) > 0 \Rightarrow f(c)$ IS LOCAL MIN

NOTICE ALREADY KNEW THIS FROM FIRST
DERIVATIVE TEST. REAL IMPORTANCE
OF f'' IS CONCAVITY OR SHAPE

ONE LAST THING

WHAT IS $\boxed{\lim_{x \to \infty} \frac{x^2-9}{x^2+3} ?}$

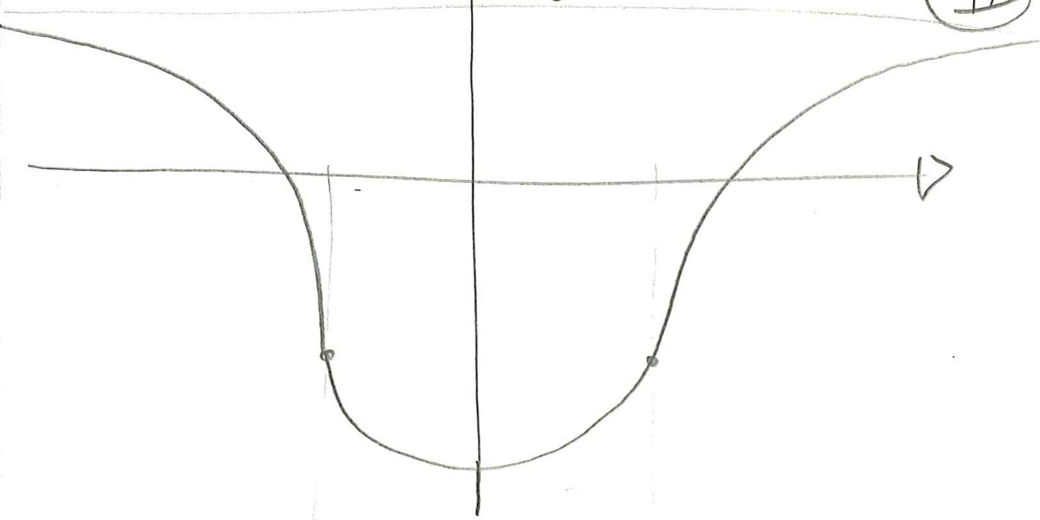
$$\lim_{x \to +\infty} \frac{x^2-9}{x^2+3} = \lim_{x \to +\infty} \frac{\cancel{x^2}(1-9/x^2)}{\cancel{x^2}(1+3/x^2)}$$

$$= \lim_{x \to +\infty} \frac{(1-\overset{\to 0}{9/x^2})}{(1+\underset{\to 0}{3/x^2})} = \frac{1}{1} = 1$$

SIMILARLY $\lim_{x \to -\infty} \frac{x^2-9}{x^2+3} = 1$

WHAT DOES THIS $y=1$ MEAN?

IX



HORIZONTAL ASSYMPTOTE! MORE NEXT TIME

[Q] SKETCH GRAPH OF f WITH

- DOMAIN $(-\infty, \infty)$
- $f(0) = 0$, $f(-1) = 1$
- $f'(x) < 0$ ON $(-\infty, +\infty)$

  ~~$f'(x) > 0$ ON $(0, +\infty)$~~
- $f''(x) > 0$ ON $(-\infty, -1)$

  $f''(x) < 0$ ON $(-1, 0)$

  $f''(x) > 0$ ON $(0, +\infty)$

TYPICAL EXAM QUESTION