MIDTERM WEDNESDAY 6:30 PM (I)

↳ SAME ROOM AS LAST TIME

OFFICE HOURS

TODAY ? $\boxed{1:30 \to 2:30}$ IN MLC

WEDNESDAY : $\boxed{1 \to 2}$

WILL RETURN QUIZZES AT END OF CLASS

& NO CLASS ON WEDNESDAY

[Q] YOU BUY A BOX OF STRAWBERRIES ON TUESDAY AND SEAL IT IN A TUPPERWARE. BY THURSDAY, YOU OBSERVE THAT THEY ARE STARTING TO MOLD & YOU ESTIMATE THAT THERE ARE 100,000 SPORES. THE MOLD SPREADS. BY FRIDAY, YOU ESTIMATE 1 MILLION SPORES. GIVEN THAT THE AMOUNT OF SPORES GROWS EXPONENTIALLY, HOW MANY SPORES DO YOU EXPECT TO FIND BY SATURDAY?

UNDERSTAND

- WHAT IS KNOWN?
- WHAT DO YOU WANT TO FIND?

KNOWN INFO: $P(t) = P_0 e^{kt}$

P_0: P(0)

k: RELATIVE GROWTH RATE

SAY COUNT t IN DAYS STARTING ON THURSDAY, THEN

$$P(0) = 100{,}000$$
$$P(1) = 1{,}000{,}000 = P_0 e^{k}$$

UNKNOWN INFO: $P(2) = ?$

$$= P_0 e^{k(2)}$$

HOW ARE WE GOING TO FIND IT?

- NEED k

$$P(1) = 1{,}000{,}000 = P_0 e^{k} = 100{,}000 e^{k}$$

$$\Rightarrow 10 = e^{k}$$

$$\Rightarrow \ln(10) = \ln(e^{k}) = k$$

$$\Rightarrow \boxed{\ln(10) = k}$$

THEREFORE

$$P(2) = 100{,}000\, e^{\ln(10)\, 2}$$

LOG RULE!

$$= 100{,}000\, e^{\ln(10^2)}$$

$$= 10{,}000{,}000 \text{ SPORES SATURDAY}$$

WHILE WE'RE TALKING ABOUT EXPONENTS

$\hookrightarrow$ WHAT IS $h'(\pi)$ WHEN $h(x) = x^{\cos(x)}$

$$h(x) = x^{\cos(x)}$$

$$\Rightarrow \ln(h(x)) = \ln\left(x^{\cos(x)}\right) = \cos(x)\ln(x)$$

$$\Rightarrow \frac{h'(x)}{h(x)} = (-\sin(x))\ln(x) + \frac{\cos(x)}{x}$$

$$\Rightarrow \frac{h'(\pi)}{h(\pi)} = -\sin(\pi)\ln(\pi) + \frac{\cos(\pi)}{\pi}$$

$$\Rightarrow \frac{h'(\pi)}{\pi} = 0 + \frac{1}{\pi} = \frac{1}{\pi}$$

$$\Rightarrow \boxed{h'(\pi) = 1}$$

① IV

OBJECTIVE: $E(H,A) = \dfrac{H(100+A)}{100}$

CONSTRAINT: $3600 = 2.5H + 18A$

$\Rightarrow \dfrac{3600 - 2.5H}{18} = A$

$$\Rightarrow E(H) = \frac{H\left(100 + \left[\frac{3600-2.5H}{18}\right]\right)}{100}$$

$$= H + \frac{3600}{1800}H - \frac{2.5}{1800}H^2$$

$$= H + 2H - \frac{1}{720}H^2 = 3H - \frac{1}{720}H^2$$

DOMAIN: $0 \le H \le \dfrac{3600}{2.5} = 1440$

OPTIMIZE $E'(H) = 3 - \dfrac{2}{720}H = 3 - \dfrac{1}{360}H$

SINGLE CRITICAL POINT

WHEN $0 = 3 - \dfrac{1}{360}H \Rightarrow \boxed{H = 1080}$

COMPARE VALUES (~~EXTREME~~ CLOSED INTERVAL METHOD) V

$E(0) = 0$

$E(1440) = 3(1440) - \frac{(1440)^2}{720} = 1440$

NOT OBV

$E(1080) = 3(1080) - \frac{(1080)^2}{720} = 1620$

WHAT ELSE COULD WE DO?

INCREASING / DECREASING TEST

INTERVAL	E'	E
$(-\infty, 1080)$	POSITIVE	$\nearrow$
$(1080, +\infty)$	NEGATIVE	$\searrow$

$\Rightarrow$ ABSOLUTE MAX

A FUNCTION WITH A SINGLE CRITICAL

THEN LOCAL MAX $\Rightarrow$ ABSOLUTE MAX

LOCAL MIN $\Rightarrow$ ABSOLUTE MIN

(2) VI

CONSTRAINT: $1800 = 2.5H + 18A$

AMOUNT YOU BUY

OBJECTIVE:

$$E(H,A) = \frac{(2000+H)(100+[50+A])}{100}$$

FROM CONSTRAINT $\left(\frac{1800-2.5H}{18}\right) = A$

$$\Rightarrow E(H) = \frac{(2000+H)\left(150+\left[\frac{1800-2.5H}{18}\right]\right)}{100}$$

$$= \frac{1}{100}(2000+H)\left(250 - \frac{2.5}{18}H\right)$$

DOMAIN: $0 \leq H \leq \frac{1800}{2.5}$

$$\Rightarrow E'(H) = \frac{1}{100}\left[\left(250-\frac{2.5}{18}H\right) + (2000+H)\left(\frac{-2.5}{18}\right)\right]$$

ONLY ONE CRIT

$\Rightarrow H = -100$ OUTSIDE DOMAIN

$\Rightarrow$ MAX WILL HAPPEN? AT ENDPOINT

$E(0) = \frac{50\,000}{100} = 5000$

$E(720) = 4080$

SPEND ALL MONEY ON ARMOR