- OFFICE HOURS
  - TODAY 1:30 → 2:30
  - TOMORROW 2:00 → 3:00
  } IN MLC
- QUIZZ ON FRIDAY
- MIDTERM WEDNESDAY NOVEMBER 12 @ 6:30
  - COMPUTE DERIVATIVES
    (EX: CHAIN RULE, LOGARITHMIC, EXPONENTIAL IMPLICIT DIFFERENTIATION)
    & USE THEM (EX SLOPE TAN LINE MAX/MIN)
  - SKETCHING GRAPHS
  - RELATED RATES
  - PRICE ELASTICITY OF DEMAND
  - EXPONENTIAL MODELS
    (EX BACTERIA, CONTINUOUS COMPOUNDING GROWTH)

[Q] YOU ARE GIVEN A CTS f FOR WHICH $f''(x) > 0$ ON $(-\infty, a) \cup (a, +\infty)$

T/F f MIGHT HAVE AN ABSOLUTE MAX AT a

OPTIMISATION

OUR LONG LOST GOAL

REAL WORLD PROBLEM → NEED TO MAXIMIZE OR MINIMIZE SOMETHING → USE CALCULUS

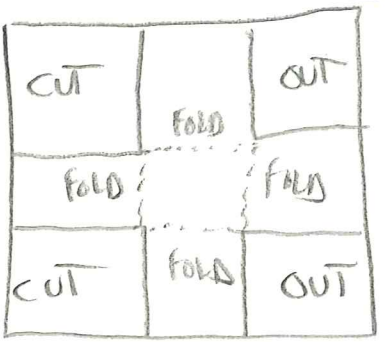
HARD STEP

EASY STEP

TODAY FOCUS ON FINDING THE FUNCTION WE NEED TO OPTIMIZE

TYPICAL EXAMPLE

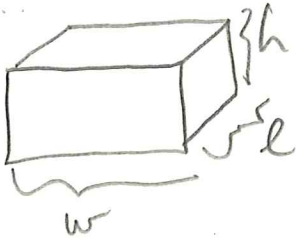
GIVEN A SQUARE OF CARDBOARD (12×12) YOU WISH TO MAKE AN OPEN BOX OF LARGEST POSSIBLE VOLUME BY CUTTING OUT SQUARES FROM FOUR CORNERS OF YOUR SHEET TO FOLD IT INTO A BOX.

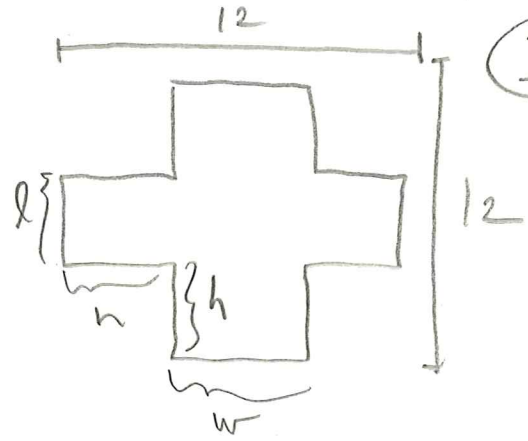


HOW BIG SHOULD THE CUT-OUT SQUARES BE?

WHAT DOES YOUR INTUITION SAY!?

WAIT





III

$V = w \cdot l \cdot h$ WHAT NEEDS TO BE MINIMIZED

↳ MUST USE INFO TO MAKE INTO A FUNCTION OF A SINGLE VARIABLE (SO CAN USE CALCULUS)

SINCE WE CUT OUT SQUARES

$l = w = 12 - 2h$ ← CONSTRAINT

↳ $V(h) = (12-2h)(12-2h)h = 144h - 48h^2 + 4h^3$

DOMAIN? $0 \leq h \leq 6$

MAXIMIZE $V(h)$ ON $[0,6]$

$V'(h) = 144 - 96h + 12h^2 = 12(h-6)(h-2)$

TWO CRIT POINTS $h=6$ & $h=2$

COMPARE & CONTRAST

IV

$V(2) = 128$
$V(6) = 0$
} CRITS

$V(6) = 0$
$V(0) = 0$
} END POINTS

} ⇒ ABSOLUTE MAX VOLUME IS 128

Q IS THIS OUR ANSWER !?

NO!

ANSWER IS WE SHOULD CUT OUT 2x2 SQUARES

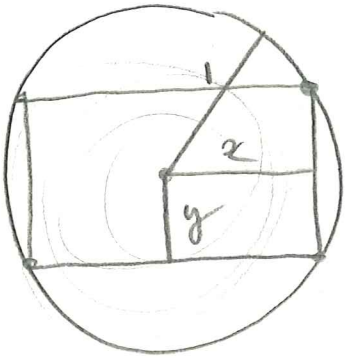
WHAT WERE OUR STEPS ?

- UNDERSTAND THE PROBLEM
  - READ IT TWICE!
  - DRAW A PICTURE
  - USE VARIABLES TO DESCRIBE QUANTITIES
  - EXPRESS RELATIONSHIPS BETWEEN VARIABLES BY EQUATIONS
  - USE THESE EQUATIONS TO EXPRESS THE QUANTITY OF INTEREST AS FUNCTION OF A SINGLE VARIABLE
- SOLVE THE PROBLEM
  - USE CALCULUS TO MAX/MIN
- ANSWER ORIGINAL QUESTION
  - REREAD PROBLEM!

SAW MILL PROBLEM

V

SUPPOSE NEED TO CUT A BEAM WITH MAXIMAL RECTANGULAR CROSS-SECTION FROM A CIRCULAR LOG OF RADIUS 1 FT, WHAT ARE CROSS-SECTIONAL DIMENSIONS OF SUCH A BEAM? WAIT



① $A = (2x)(2y) = 4xy$

② $x^2 + y^2 = 1$

$\Rightarrow y = \sqrt{1-x^2}$

WHY (+)?

COMBINE $\boxed{A(x) = 4x\sqrt{1-x^2}}$

DOMAIN $0 \le x \le 1$ ⇝ MAXIMIZE

ANSWER WILL NOT BE MAX BUT "x"

↳ DO IT YOURSELF AND FIND A SQUARE

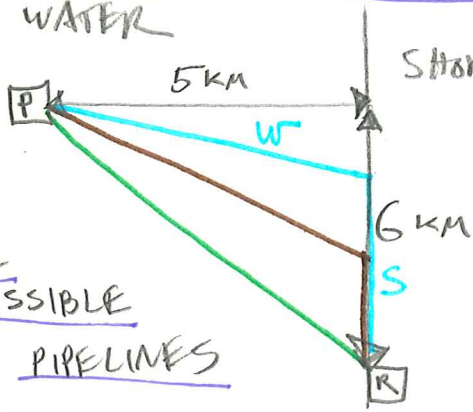
VI

OFFSHORE OIL PLATFORM MUST BE JOINED TO REFINERY ON LAND BY A PIPELINE. IF THE COST OF BUILDING A PIPELINE UNDER WATER IS 10^5 $/KM, MEANWHILE THE COST OF BUILDING A PIPELINE ON LAND IS 10^4 $/KM, IF THE PLATFORM IS 5 KM OFFSHORE AND THE REFINERY IS 6 KM SOUTH OF THE POINT ON THE SHORE CLOSEST TO THE PLATFORM, WHAT IS THE LEAST POSSIBLE COST OF THIS PIPELINE?

(ASSUME THE SHORE LINE IS STRAIGHT NORTH-SOUTH & THAT PIPELINE CONSISTS OF AT MOST TWO STRAIGHT SEGS)

TALK TO EACH OTHER FIRST

VII



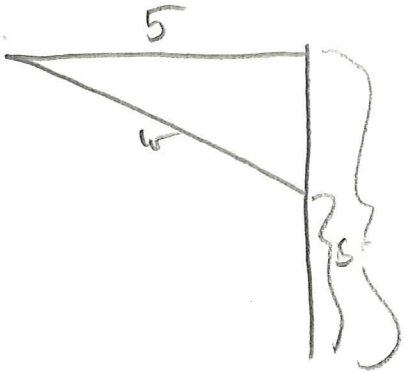
THREE POSSIBLE PIPELINES

W DISTANCE UNDER WATER

S DISTANCE ON SHORE

COST $C = 10^5 W + 10^4 S$

How to MAKE FUNCTION OF SINGLE VARIABLE ? FIND CONSTRAINT



$5^2 + (6-S)^2 = W^2$

$\Rightarrow \sqrt{5^2+(6-S)^2} = W$

SO $C(S) = 10^5 \sqrt{5^2+(6-S)^2} + 10^4 S$

MUST BE MINIMIZED

DOMAIN? $0 \leq S \leq 6$ $[0, 6]$