COURSE WEBSITE

↳ PAST FINAL EXAMS WITH HINTS AND SOLUTIONS

↳ THOMPSON 2 OF 2 (ANOTHER SAMPLE MIDTERM)

GOOD PRACTISE FOR

MIDTERM WEDNESDAY NOVEMBER 12

@ 6:30 PM IN GEOG 200

LAST TIME: OPTIMISATION

- UNDERSTAND PROBLEM
  - ↳ PICTURE
  - ↳ VARIABLES
  - ↳ CONSTRAINTS (i.e EQUATIONS)
  - ↳ IDENTIFY OBJECTIVE FUNCTION (THE ONE THAT NEEDS TO BE MAXIMIZED / MINIMIZED)
- SOLVE PROBLEM
  - ↳ CALCULUS
- ANSWER ORIGINAL QUESTION

HARD TO "LEARN"

TAKES PRACTISE

[Q 2011 FINAL]

II

FIND TWO POSITIVE REAL NUMBERS $m \geq 1$ & $n \geq 1$ WHOSE PRODUCT IS 50 AND WHOSE SUM IS AS SMALL AS POSSIBLE.

CONSTRAINT: $m \cdot n = 50 \Rightarrow m = \frac{50}{n}$

OBJECTIVE FUNCTION: $S = m + n$

↳ SINGLE VARIABLE $\boxed{S(n) = \frac{50}{n} + n}$

DOMAIN: $1 \leq n \leq 50$

USE CALCULUS: $S'(n) = \frac{-50}{n^2} + 1$

CRITICAL POINTS: $\frac{50}{n^2} = 1 \Rightarrow \sqrt{50} = n$

END POINTS: 1, 50

$S(1) = 50 + 1 = 51$

$S(50) = 1 + 50 = 51$

$S(\sqrt{50}) = \frac{50}{\sqrt{50}} + \sqrt{50}$

$= \sqrt{50} + \sqrt{50} = 2\sqrt{50}$

WHAT IS BIGGER? ESTIMATE! III

$7^2 = 49$ $\quad$ $8^2 = 64$

$\Rightarrow \sqrt{50} \approx 7$ $\quad$ $\Rightarrow 2\sqrt{50} \approx 14 \ll 51$

SO $n = \sqrt{50}$ & $m = \dfrac{50}{n} = \dfrac{50}{\sqrt{50}} = \sqrt{50}$

IS ANSWER!

[Q] YOU HAVE A PIECE OF WIRE OF LENGTH L FROM WHICH YOU CONSTRUCT A CIRCLE AND/OR A SQUARE IN SUCH A WAY THAT THE TOTAL ENCLOSED AREA IS MAXIMAL. THEN YOU SHOULD CONSTRUCT

(a) ONLY THE SQUARE

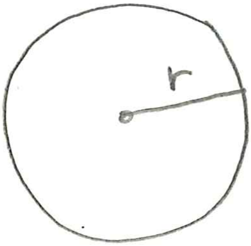
(b) ONLY THE CIRCLE

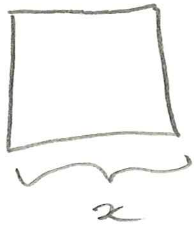
(c) BOTH BUT PERIMETER (□) > PERIMETER (○)

(d) BOTH BUT PERIMETER (□) < PERIMETER (○)

DRAW & USE INTUITION FIRST

IV





REASON GEOMETRICALLY FIRST.

CONSTRAINT: $\underline{2\pi r + 4x = L}$

OBJECTIVE: $\underline{A = \pi r^2 + x^2}$

REDUCE OBJECTIVE FUNCTION TO SINGLE VARIABLE USING CONSTRAINT

$$\underline{2\pi r = L - 4x}$$

$$\boxed{r = \frac{L-4x}{2\pi}} \Rightarrow \boxed{A(x) = \pi\left(\frac{L-4x}{2\pi}\right)^2 + x^2}$$

DOMAIN: $\underline{0 \leq x \leq \frac{L}{4}}$

$$A'(x) = \pi(2)\left(\frac{L-4x}{\cancel{2\pi}}\right)(-4) + 2x$$

$$A'(x) = -4L + 16x + 2x = -4L + 18x$$

CRIT POINTS: $\underline{4L = 18x}$

$$\Rightarrow \boxed{\frac{2L}{9} = x}$$

V

ENDPOINTS: $x=0$

$x=\frac{L}{4}$

COMPARE: $A(0)=\pi\left(\frac{L}{2\pi}\right)^2=\boxed{\frac{L^2}{4\pi}}$

$$A\left(\frac{L}{4}\right)=\pi\left(\frac{L-4\left(\frac{L}{4}\right)}{2\pi}\right)^2+\left(\frac{L}{4}\right)^2$$

$$=\boxed{\frac{L^2}{16}}$$

$$A\left(\frac{2L}{9}\right)=\pi\left(\frac{L-4\left(\frac{2L}{9}\right)}{2\pi}\right)^2+\left(\frac{2L}{9}\right)^2$$

$$=\frac{\left(L-\frac{8}{9}L\right)^2}{4\pi}+\frac{4L^2}{81}$$

$$=\frac{\left(\frac{1}{9}L\right)^2}{4\pi}+\frac{4L^2}{81}$$

$$=\frac{L^2}{(4\pi)(81)}+\frac{4L^2}{81}=\left(\frac{1}{(4\pi)81}+\frac{(4\pi)4}{(4\pi)81}\right)L^2$$

$$=\boxed{\left(\frac{1+16\pi}{(4\pi)81}\right)L^2}$$

VI

$\frac{1}{4\pi}L^2$	$\frac{1}{16}L^2$	$\frac{1+16\pi}{(4\pi)(81)}L^2$

WHO IS BIGGER?

$\pi < 4$

SO $\frac{1}{4\pi} > \frac{1}{4\cdot 4} = \frac{1}{16} \Rightarrow \frac{1}{4\pi}L^2 > \frac{1}{16}L^2$

OK WHAT ABOUT NASTY GUY?

$$\frac{1}{4\pi} = \frac{81}{(4\pi)(81)} \text{ VS } \frac{1+16\pi}{(4\pi)(81)}$$

WELL $\pi < 4$

SO $1+16\pi < 1+16(4) = 1+64 = 65 < 81$

WE CONCLUDE ABSOLUTE MAX IS AT $\boxed{x=0}$ WITH $\boxed{A(x) = \frac{1+16\pi}{(4\pi)(81)}L^2}$

JUST CONSTRUCT THE CIRCLE!