WHERE DO WE COME FROM? (I)

WHERE ARE WE HEADING?

- NEED TO UNDERSTAND THE "WORLD AROUND US" (ECON, BIOLOGY, SPACE, PERSPECTIVE IN ART, LINGUISTICS)
- REQUIRES AN ABILITY TO DESCRIBE COMPLICATED RELATIONSHIPS BETWEEN COMPONENTS OF THIS WORLD
- MATHEMATICS IS A UNIVERSAL LANGUAGE BUILT TO EXPLAIN SUCH RELATIONSHIPS IN A SIMPLE MANNER

FOCUS ON DIFFERENT PARTS DURING THE TERM

SYSTEM

RELATIONSHIPS

FUNCTIONS

CALCULUS

USUALLY DIFFERENTIABLE FOR MOST PART

EVEN DIFF FUNCTIONS CAN BE HARD TO HANDLE DIRECTLY

II

↳ $f(x) = \sqrt{x} \rightsquigarrow f(3) = ?$

(YOUR COMPUTER DOESN'T EVEN KNOW!)

IDEA OF CALCULUS

KNOWING A LITTLE ABOUT $f'(x)$ → TELLS YOU A LOT ABOUT $f(x)$

SAW THIS IN CONTEXT OF SKETCHING GRAPHS ...

BUT SOMETIMES NEED TO CRUNCH SOME ACTUAL NUMBERS.

THIS IS DONE BY APPROXIMATING DIFF FUNCTIONS AS POLYNOMIALS

↳ LINEAR APPROXIMATION

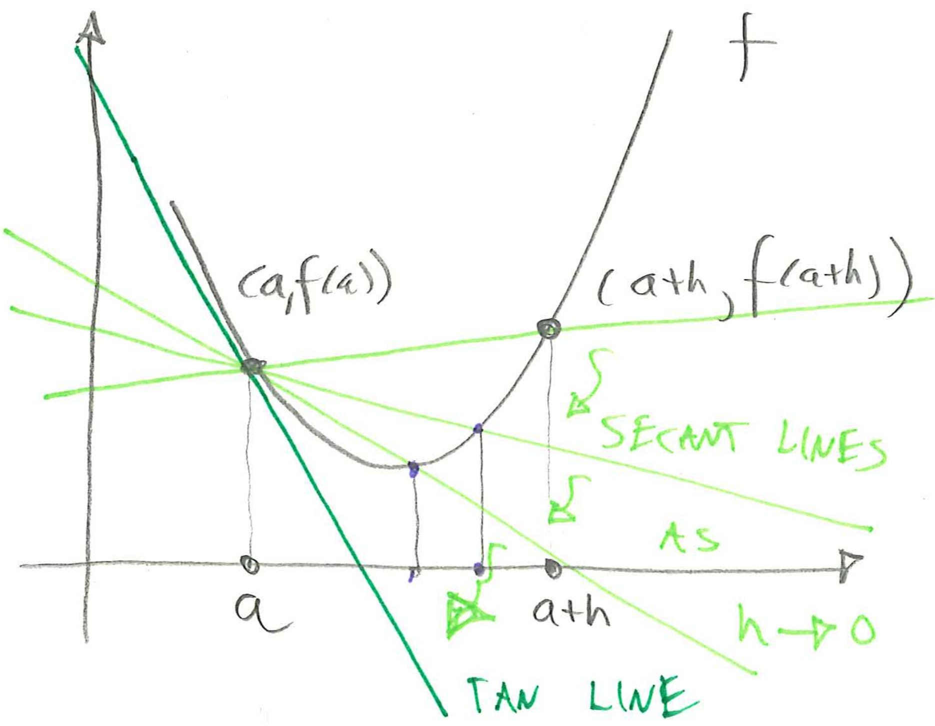
↳ TAYLOR EXPANSION (WHAT COMPUTERS USE)

GO BACK TO THE SOURCE: III

WHAT IS THE DERIVATIVE OF f AT a?

$f'(a)$ = SLOPE OF TAN LINE TO GRAPH OF f AT $(a, f(a))$

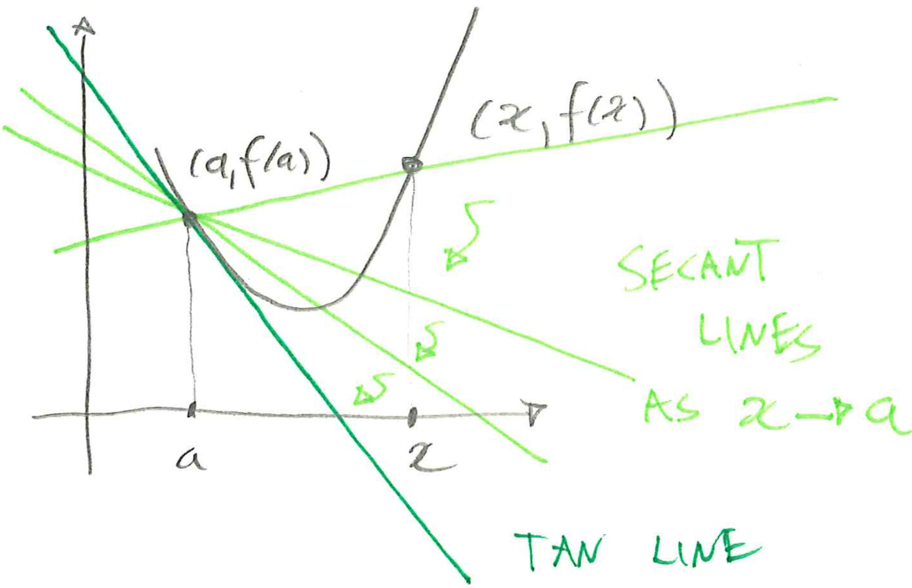
= LIMIT OF SLOPES OF SECANTS THROUGH $(a, f(a))$ & $(a+h, f(a+h))$ AS $h \to 0$



WRITING "$a+h$" IS JUST A WAY OF SAYING "A NUMBER CLOSE TO a"

LET'S CALL THIS NUMBER x

ie LET $x = a+h$



$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

TWO WAYS OF WRITING SAME THING

LIMIT DEF SAYS

CAN MAKE $\dfrac{f(x)-f(a)}{x-a}$ AS CLOSE AS WE LIKE TO $f'(a)$ BY REQUIRING x TO BE SUFFICIENTLY CLOSE, BUT UNEQUAL TO, a

AKA IF x VERY CLOSE TO a

THEN $\dfrac{f(x)-f(a)}{x-a} \approx f'(a)$

APPROXIMATELY

REARRANGE $\boxed{f(x)-f(a) \approx f'(a)(x-a)}$

LOOKS LIKE POINT-SLOPE FORM OF THE EQUATION OF A LINE

$$y - y_0 = m(x - x_0) \ !$$

DEFINE A LINEAR FUNCTION $L(x)$

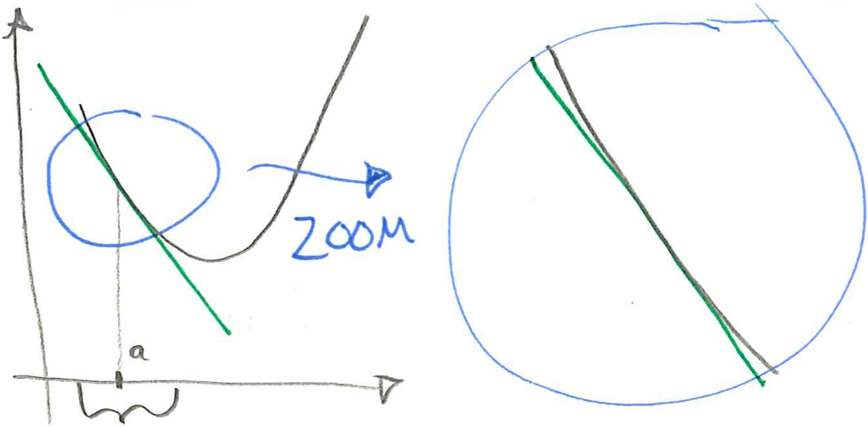
$$\boxed{L(x) - f(a) = f'(a)(x-a)}$$

FUNCTION CONSTANTS VARIABLE

$$\boxed{L(x) = f(a) + f'(a)(x - a)}$$ VI

Q: WHAT DOES THE GRAPH OF ↑ LOOK LIKE?

↳ IT'S THE TANGENT LINE



↳ WHEN x IS VERY CLOSE TO a VALUES OF $f(x)$ AND $L(x)$ ARE ALMOST THE SAME.

Q: WHY IS THIS USEFUL?

GIVEN a, $f(a)$ & $f'(a)$ IT IS VERY EASY TO COMPUTE $L(x)$!

EX $\sqrt{4} \stackrel{?}{=} 2$ DUH

OK WHAT ABOUT $\sqrt{4.1}$?

4 & 4.1 ARE VERY CLOSE

SO CAN USE LINEAR APPROX FOR

$f(x) = \sqrt{x}$ AT THE POINT 4

WRITE

$$\boxed{L(x) = f(4) + f'(4)(x-4)}$$

$f(4) = \sqrt{4} = 2$ ✓

$f'(x) = \dfrac{1}{2\sqrt{x}}$

SO $f'(4) = \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

$\Rightarrow L(x) = 2 + \frac{1}{4}(x-4)$

EVALUATE AT $x = 4.1$

$L(4.1) = 2 + \frac{1}{4}(4.1-4)$

$= 2 + \frac{1}{4}(0.1) = 2 + \frac{1}{40} = 2.025$

PRETTY CLOSE INDEED

TRUE VALUE $f(4.1) = \sqrt{4.1} = 2.0248\ldots$

Ex) $\ln(1) \overset{?}{=} 0$ DUH

OK WHAT ABOUT $\ln(1.1)$?

$g(x) = \ln(x)$ KNOW $g(1) = \ln(1) = 0$

KNOW $g'(x) = \frac{1}{x} \rightarrow g'(1) = \frac{1}{1} = 1$

SINCE 1 & 1.1 ARE VERY CLOSE

CAN USE LINEAR APPROX OF g AT 1

TO ESTIMATE $g(1.1)$

WRITE

$$L(x) = g(1) + g'(1)(x-1)$$
$$= 0 + 1(x-1)$$

$$\boxed{L(x) = x-1}$$

$\Rightarrow L(1.1) = 1.1 - 1 = 0.1$

PRETTY CLOSE

TRUE VALUE $g(1.1) = \ln(1.1) = 0.095\ldots$

HOMEWORK!