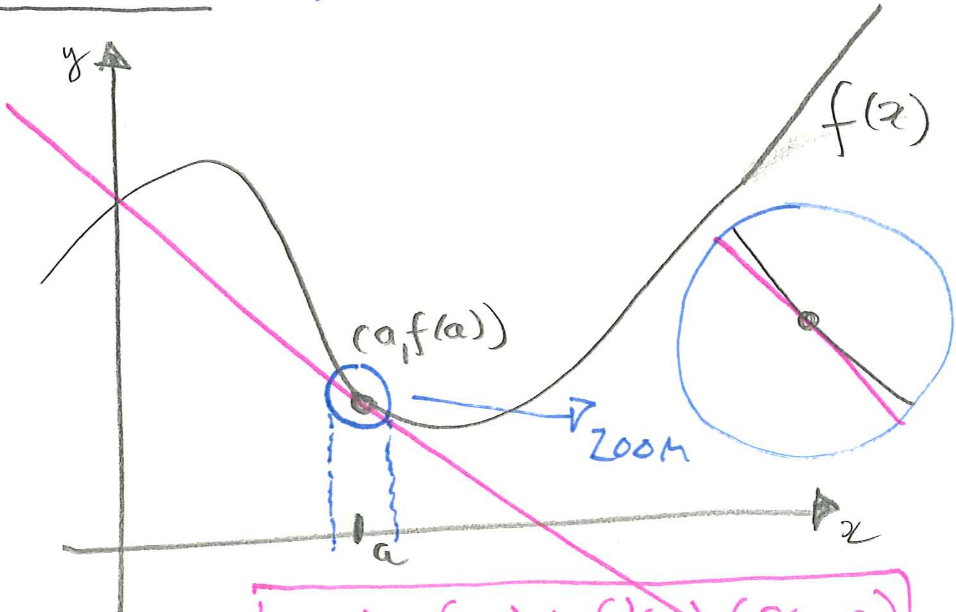
I WILL RETURN / GO OVER MIDTERM @ END (I)

LAST TIME: LINEAR APPROXIMATION



$$\boxed{L(x) = f(a) + f'(a)(x-a)}$$

USING LIMIT DEFINITION OF $f'(a)$ WE SAW THAT $\boxed{f(x) \approx f(a) + f'(a)(x-a)}$

ie: THE LINE $L(x)$ HAS VALUES VERY CLOSE TO $f(x)$ WHENEVER x IS VERY CLOSE TO a.

WHY DO WE CARE? BECAUSE IT IS EASIER TO COMPUTE VALUES $L(x)$!

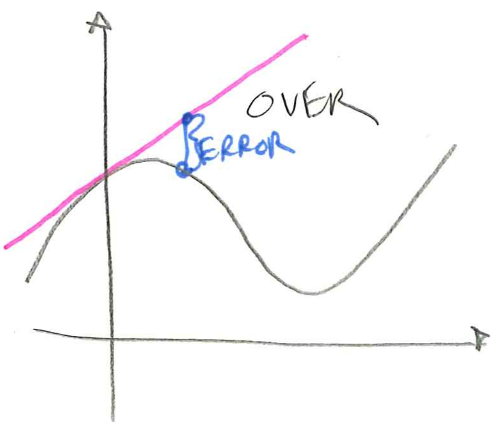
[Q] HOMEWORK FOLLOW UP

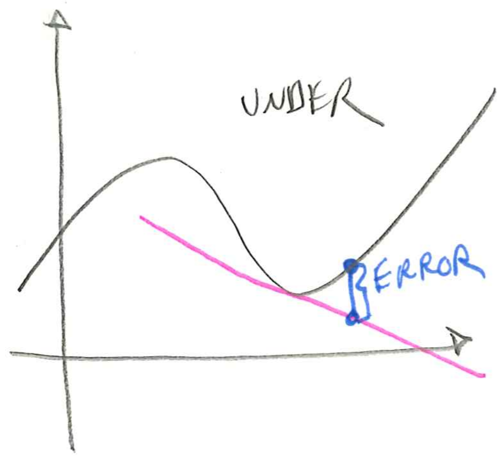
SUPPOSE $f''(x) < 0$ FOR x NEAR A POINT a. THEN THE LINEAR APPROX OF f AT a IS

Ⓐ AN OVERESTIMATE

Ⓑ AN UNDERESTIMATE

Ⓒ NOT ENOUGH INFO GIVEN





RECALL:

CONCAVE DOWN

$f' \downarrow \Rightarrow f'' < 0$

GRAPH TRAPPED BELOW TANGENTS

CONCAVE UP

$f' \uparrow \Rightarrow f'' > 0$

GRAPH TRAPPED ABOVE TANGENTS

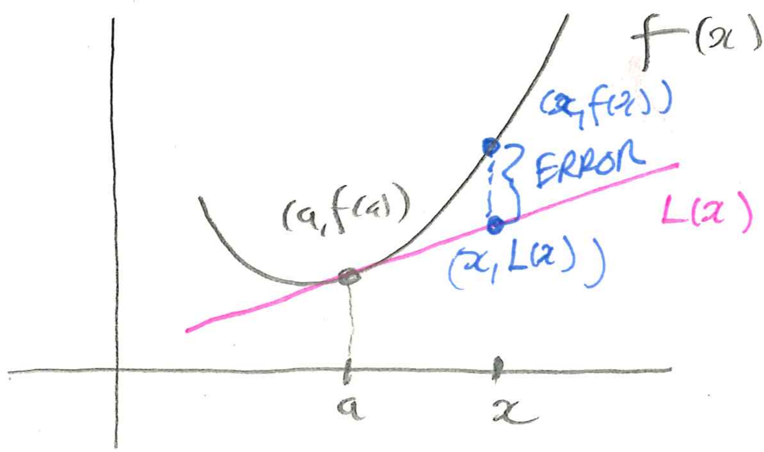
III

BUT WHAT $\boxed{\text{IS}}$ THE ERROR?

$f(x)$ vs $L(x) = f(a) + f'(a)(x-a)$

ERROR AT x IS

$$R(x) = f(x) - L(x)$$
$$= f(x) - f(a) - f'(a)(x-a)$$



$R(x)$ IS A FUNCTION OF x

BECAUSE? BIG VS SMALL DEPENDING ON HOW FAR FROM ANCHOR YOU ARE

CAN WE SAY SOMETHING MORE PRECISE?

$\boxed{\text{Q}}$. $\lim_{x \to a} R(x) = ?$ 0

WHAT DOES THIS MEAN?

IV

INTERLUDE

$$\lim_{x \to 0} x = 0 \qquad \lim_{x \to 0} x^2 = 0$$

$$\lim_{x \to 0^+} \frac{1}{x} = +\infty \qquad \lim_{x \to 0^+} \frac{1}{x^2} = +\infty$$

$$\lim_{x \to 0} \frac{x^2}{x} = \lim_{x \to 0} \frac{x}{1} = 0$$

↳ TOP & BOTTOM GO TO ZERO

BUT TOP GOES TO ZERO FASTER

$$\lim_{x \to 0^+} \frac{x}{x^2} = \lim_{x \to 0^+} \frac{1}{x} = +\infty$$

↳ TOP & BOT GO TO ZERO

BUT BOT GOES TO ZERO FASTER

HOW FAST DOES $R(x) \to 0$ AS $x \to a$?

NOTICE: THIS IS ANOTHER WAY OF ASKING HOW CLOSE TO a WE MUST BE TO GET A GOOD APPROX

[Q] RECALL

$$R(x) = f(x) \underbrace{- f(a) - f'(a)(x-a)}_{-L(x)}$$

WE ALREADY SAW $\lim_{x \to a} R(x) = 0$

WHAT ABOUT $\lim_{x \to a} \frac{R(x)}{(x-a)}$?

(A) MUST BE 0

(B) MIGHT NOT EXIST (e.g. $= +\infty$)

(C) DEPENDS ON THE VALUE OF a.

↳ GO AHEAD, COMPUTE THE LIMIT!

SPLIT & CONQUER

$$\lim_{x \to a} \frac{f(x) - f(a) - f'(a)(x-a)}{x-a}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x-a} - \lim_{x \to a} \frac{f'(a)(x-a)}{(x-a)}$$

$$= f'(a) - f'(a) = 0 ;$$

$\Rightarrow R(x) \to 0$ FASTER THAN

$(x-a) \to 0$ AS $x \to a$

IN FACT:

THM (ERROR OF LINEAR APPROX) VI

IF $|f''(c)| \leq M$ FOR ALL

c BETWEEN a AND x THEN

THE ERROR W THE LINEAR

APPROX TO $f(x)$ AT a

$L(x) = f(a) + f'(a)(x-a)$

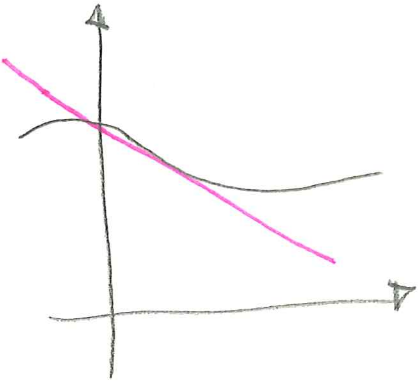
IS BOUNDED AS FOLLOWS

$$\boxed{|R(x)| \leq \frac{M}{2}|x-a|^2}$$

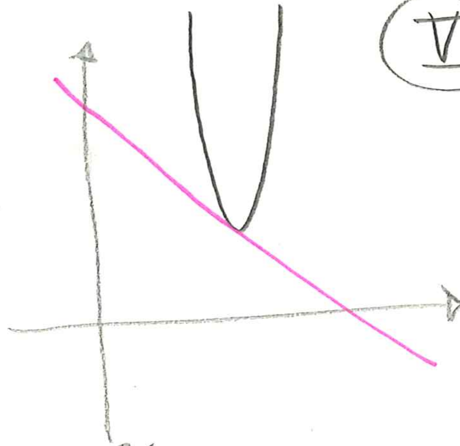
Q: WHY DOES THIS MAKE SENSE?
(CONSISTENT WITH WHAT WE OBSERVED)

↳ f'' TELLS US HOW FAST THE DERIVATIVE IS INCREASING / DECREASING

↳ CONTROLLS HOW QUICKLY GRAPH OF f CAN ESCAPE AWAY FROM GRAPH OF TANGENT LINE



VS



EXAMPLE: APPROXIMATE $(9.1)^{3/2}$ & GIVE A BOUND ON ERROR OF APPROX

$f(x) = x^{3/2}$

$f'(x) = \frac{3}{2} x^{1/2}$

$f''(x) = \frac{3}{4} x^{-1/2}$

$f(9) = 9^{3/2} = (\sqrt{9})^3 = 27$

$\underbrace{f'(9) = 9/2}_{\text{9 IS A GOOD ANCHOR}}$

$L(x) = f(9) + f'(9)(x-9)$

$L(x) = 27 + \frac{9}{2}(x-9)$

$L(9.1) = 27 + \frac{9}{20} = 27.45$ ← HOW ACCURATE?

$|R(x)| \leq \frac{1}{2} M |x-a|^2$ ⇝ WANT $R(9.1)$

IX

#1 a) BASIC CHAIN RULE

b) BASIC LOG DIFF

c) BASIC IMPLICIT

#2 BUSINESS PROBLEM

(EASIER THAN QUIZ)

#3 DONE IN CLASS

#4 BASIC SKETCH

#5 BASIC ASYMPTOTE

& MAX / MIN PROBLEM

POINTS GOT ADDED

#6 HARD

- MADE A FEW CHANGES TO MARKS

- ON FINAL USE LIM TO FIND ASYMPTOTES