SAMPLE FINAL BREAKDOWN

8:30 AM
SEC A-B-C 6th

I

SHORT (42%) VS LONG (58%)

QUESTIONS

- LIMITS
- EXPONENTIAL MODELS
- CHAIN RULE
- FIND TANGENT LINES
- DEF$^{\underline{n}}$ OF DERIVATIVE
- CURVE SKETCHING
- OPTIMIZATION
- TAYLOR POLYNOMIAL

BUSINESS
PROBLEM
LANGUAGE

10% • DEF$^{\underline{n}}$ OF DERIVATIVE

BIG CHUNK

14% • CURVE SKETCHING
15% • OPTIMIZATON
12% • RELATED RATES

$\approx$ 40%

7% • TAYLOR APPROX

THINK ABOUT YOUR STRATEGY

• TAKE FIRST 5 MINUTES TO LOOK OVER EXAM & CHOOSE WHAT YOU WILL DO FIRST (II)

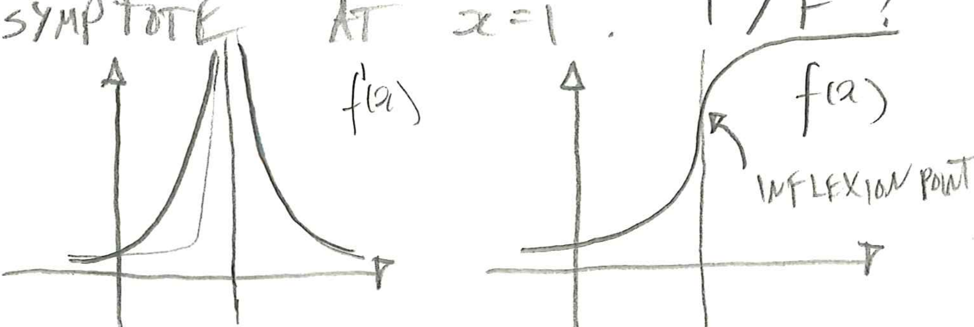
• ON YOUR FIRST RUN: → ≈ 5 MINS
SHORT

THEN CYCLE THROUGH → ≈ 15 MINS
LONG

• PLENTY OF TIME THOUGH
WORK SLOWLY & CAREFULY

↳ LEARNING GOALS → COMPLETE LIST OF TOPICS
↳ DO THE SAMPLE FINAL (WITH TIME PRESSURE)
↳ USE MATH EXAM RESSOURCE WIKI
STAY ZEN, HYDRATE, SLEEP > STUDY, ETC.

Q IF $f'(x)$ HAS A VERTICAL ASYMPTOTE AT $x=1$ THEN $f(x)$ MUST HAVE A VERT ASYMPTOTE AT $x=1$. T / F ?



Q DOES $f(x) = e^x - 2x^2 + 1$ HAVE ANY CRITICAL POINTS? III

(A) NO (B) YES @ $x=0$ (C) YES @ $x=1$ (D) YES @ ?
(E) DON'T KNOW

$f'(x) = e^x - 4x$ EXISTS EVERYWHERE & IS EQUAL TO ZERO WHEN...

↳ CTS!

↳ CAN USE IVT → WHAT DOES IT SAY?

$f'(0) = e^0 - 4(0) = 1 > 0$

$f'(1) = e^1 - 4 = e - 4 < 0$ WHY?

⇒ ∃ CRIT BETWEEN 0 & 1

Q EVALUATE $\lim_{x \to 1} \frac{\ln(x)}{x-1}$

$$\lim_{x \to 1} \frac{\ln(x)}{x-1} = \lim_{x \to 1} \frac{\ln(x) - 0}{x-1} = \lim_{x \to 1} \frac{\ln(x) - \ln(1)}{x-1}$$

$= f'(1)$ WHERE $f(x) = \ln(x)$

$= \frac{1}{1} = 1$ ✓

Q WHAT IS MAX # OF HORIZONTAL ASYMPOTES A FUNCTION CAN HAVE? IV

DEF^{n} f HAS A HORIZONTAL ASYMPTOTE AT $+\infty$ IF $\lim_{x \to \infty} f(x) = L$

Q WHAT IS MAX # OF VERTICAL ASYMPTOTES A FUNCTION CAN HAVE?

↳ GRAPH OF TAN & REPEAT

Q CAN A FUNCTION CROSS ONE OF ITS HORIZONTAL ASYMPTOTES? VERTICAL ASYMPTOTES?

↳ PRACTISE CURVE SKETCHING!

Q IF BASE b OF Δ IS INCR AT 3cm/s WHILE ITS HEIGHT DECREASES @ 3cm/s then

(A) A IS INCR
(B) A IS DECR
(C) A IS DECR WHEN $b < h$
(D) A IS DECR WHEN $b > h$
(E) A REMAINS CONSTANT

$A' = \frac{1}{2}(b'h + bh')$
$= \frac{1}{2}(3h - 3b)$

So (D) ANS!

↳ PRACTISE SETTING UP RR PROBLEMS

$\boxed{Q}$ YOU BORROW 10,000 \$ FROM A SHARK (IV)
WHO CHARGES YOU A FIXED RATE r CTSLY COMPOUNDED
IF YOU PAY HIM 100,000 \$ 2 YEARS LATER, WHAT
WAS THE ANNUAL INTEREST RATE?

↳ $A = Pe^{rt}$ RATE $100{,}000 = 10{,}000\, e^{2r}$

AMOUNT   INITIAL $\Rightarrow 10 = e^{2r} \Rightarrow \ln(10) = 2r$

$\boxed{Q}$ TWO CYLINDRICAL TANKS ARE FILLED
SIMULTANEOUSLY AT EXACTLY THE SAME RATE,
THE SMALLER TANK HAS RADIUS 5 m
& WATER RISES @ RATE OF 0.5 m/min
THE LARGER TANK HAS RADIUS 8 m.
HOW FAST IS WATER RISING IN THIS ONE?

↳ WHAT DOES IT MEAN THAT THEY ARE FILLED
AT SAME RATE? $\boxed{\frac{dV_1}{dt} = \frac{dV_2}{dt}}$ V_1 = Volume of water in tank 1

$$\frac{dV_1}{dt} = \pi r_1^2 \underbrace{\frac{dh_1}{dt}}_{\text{UNKNOWN}} = \pi r_2^2 \frac{dh_2}{dt} = \frac{dV_2}{dt}$$

↳ SOLVE FOR IT → VOILÀ

REVIEW STANDARD SHADOW PROBLEMS FROM CLASS

Q A RECTANGULAR STORAGE CONTAINER (VI) WITH AN OPEN TOP IS TO HAVE VOLUME OF 8 CUBIC METRES. THE LENGTH OF ITS BASE IS TWICE THE WIDTH. MATERIAL FOR THE BASE COSTS \$4.50 / m^2 MATERIAL FOR SIDES COSTS \$6 / m^2. FIND THE COST TO PRODUCE CHEAPEST SUCH CONTAINER.

OBJECTIVE FUNCTION

$$C = 4.50 \cdot w \cdot l + 6 \cdot (2hw + 2hl)$$

CONSTRAINTS ① $2w = l$

$$\Rightarrow C = 4.50 \cdot 2w^2 + 6(2hw + 2h(2w))$$

② $V = hwl = 8$

$\Rightarrow$ (by ①) $\quad 2hw^2 = 8 \Rightarrow \boxed{h = \frac{4}{w^2}}$

$$\Rightarrow C = 4.50 \cdot 2 \cdot w^2 + 6\left(2 \cdot \frac{4}{w^2} \cdot w + 2 \cdot \frac{4}{w^2} \cdot (2w)\right)$$

$\rightsquigarrow$ NOW OPTIMIZE!