MATH 104/184: Weeks 11 and 12 Fresh Sheet
November 17, 2013

Learning Goals

Over the last two weeks of term, we will cover Approximation (Linear, Quadratic and the more general Taylor Polynomial), with a short show-and-tell about inverse trigonometric functions. Our main emphasis will be on understanding Linear Approximation. The Linear Approximation Material is in Section 4.5, and the Taylor Polynomial material will be from Section 9.1 of Briggs Cochran. The inverse trigonometric functions are covered in section 3.9 of Briggs Cochran. You should have covered the basics of inverse trigonometric functions in Math 12, and the main content of 3.9 for you will be the derivatives of $\arcsin x$, $\arccos x$, and $\arctan x$.

Suggested problems that help build these skills are given as [section: question #s].

The specific learning goals for this week for Linear Approximation are that by the end of week 12 and review homework, you should be able to:

1. explain *linear approximation* (also known as *tangent line approximation* and the *linearization* of a function) using a series of figures like those in Section 4.5; this includes being able to relate the formula for linearization to the elements of such a picture (for example: what is the role of a, what is the role of x, where is the appropriate tangent line on the graph, where does the linear approximation appear in the picture); [4.5:2,3,4,35]

2. use linear approximations to estimate the values of functions near a given $x = a$; [4.5: 9, 10, 12, 16, 18, 22, 39, 43, 45, 47]

3. use linear approximation to approximate changes in the dependent variable given changes in the independent variable; [4.5:25, 26]

4. given the exact value, discuss the discrepancy with the linear approximation in terms of the second derivative (for example, whether it is an underestimate or overestimate); [4.5: Quick Check 1, 43]

5. analyze the worst-case error for a linear approximation of a function using a formula based on the second derivative of the function;

6. *NOTE:* We are not covering the material on differentials in section 4.5;

7. use the quadratic approximation to estimate the values of functions; [9.1: 7,9]

8. explain the difference between the 2nd order term in quadratic approximation and the worst-case error term for linear approximation;

9. find the nth degree Taylor polynomial of a given function with a given centre $x = a$; [9.1: 15,19,27,65]

10. use a Taylor polynomial to approximate the values of functions; and [9.1: 33,36,37]

11. use the table of derivatives of inverse trigonometric functions in calculations of derivatives. [3.9: 7 – 12, 20,24]

Potential Learning Approaches and Issues

1. Series are formally part of MATH 105 in term 2 (and all other CA2 courses). In MATH 104/184, we introduce Taylor polynomials, but we will focus almost entirely on the linear and quadratic approximations. We will ask you to do some worst-case error analysis for the linear approximation, but not to work with the remainder formula more generally.

2. It will be important to understand how to worth with the linear approximation and a worst-case error analysis based on $M(x-a)^2/2$ where M is an upper bound on the absolute value of the second derivative on the interval defined by x and a. Our approach for understanding this worst-case error analysis is geometric. You can use what you know about what the second derivative says about concavity to see how you might do a worst-case error analysis for the linear approximation.

3. Note that this approach is slightly different than dealing explicitly with the remainder formula in Taylor's theorem on page 596 in Section 9.1. This approach is related to the Estimate the Remainder part of 9.1 in the middle of page 597, but we will not expect you to deal with the general remainder for an nth degree Taylor polynomial.

4. In section 9.1, skip the initial paragraphs on power series. We don't cover ideas of convergence in this course. These will be covered in MATH 105.

5. It is worthwhile once you have mastered the linear approximation to focus on the quadratic approximation. This will allow you to get at the Taylor formula for the coefficients in a manageable way. You can spend some time thinking about the difference between the quadratic term of the approximation and the error term for the linear approximation.

6. Once you are comfortable with the quadratic approximation, it will not be too hard to use the more general Taylor polynomials. Be sure to work through plenty of examples here, and include our old friends e^x, $\cos x$, and $\sin x$. The general Taylor polynomial will not be emphasized on the final exam.

7. For inverse trig functions, our goal is that you be able to use a table of derivatives with inverse trig functions to compute derivatives. Those taking Math 101 or 103 (which will be a few students) next term will need this for doing inverse trig substitutions.

Suggested Problems and Assignments

Suggested Problems: This week, all suggested problems from the text are:

Chapter 4.5: Quick Checks 1 – 4; Exercises 2, 3, 4, 9, 10, 12, 16, 18, 22, 25, 26, 35, 39, 43, 45, 47.

Chapter 9.1: 1,2, 6, 7, 9, 15, 19, 27, 33, 36, 37, 65.

Chapter 3.9: 7 – 12, 20, 24.

(* means the problem is hard or is a proof-type question)

Webwork Homework: You will be asked to do Assignment10 based on linear approximation. It will be due on Wednesday, November 27th.