Midterm Exam I

October 7, 2011

No books. No notes. No calculators. No electronic devices of any kind.

Name ______________________________

Student Number ________________

Problem 1. (6 points)
For each of the three matrices A, B, C find all values of the indeterminates, such that the matrix is in reduced row echelon form.

$$A = \begin{pmatrix} 0 & 0 & 1 & y & 0 \\ 0 & x & 0 & z & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 & 0 & y & z \\ 0 & 0 & 1 & x & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & y & 0 & 0 \\ 0 & 0 & 0 & x & z \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

1	2	3	4	5	6	7	total/40

Problem 2. (6 points)
Solve the following system of linear equations.
Give the answer in **parametric vector form**.

$$\begin{array}{rcrcrcrcl} 2x_1 & + & 4x_2 & + & 2x_3 & + & 2x_4 & = & 6 \\ x_1 & + & 2x_2 & + & x_3 & & & = & 3 \\ x_1 & + & x_2 & - & x_3 & & & = & 3 \end{array}$$

Problem 3. (5 points)
Decide whether or not the vector $\vec{v}$ is in span(u_1, u_2). In other words, decide if there exist x_1 and x_2, such that $\vec{v} = x_1\vec{u}_1 + x_2\vec{u}_2$. Justify your answer.

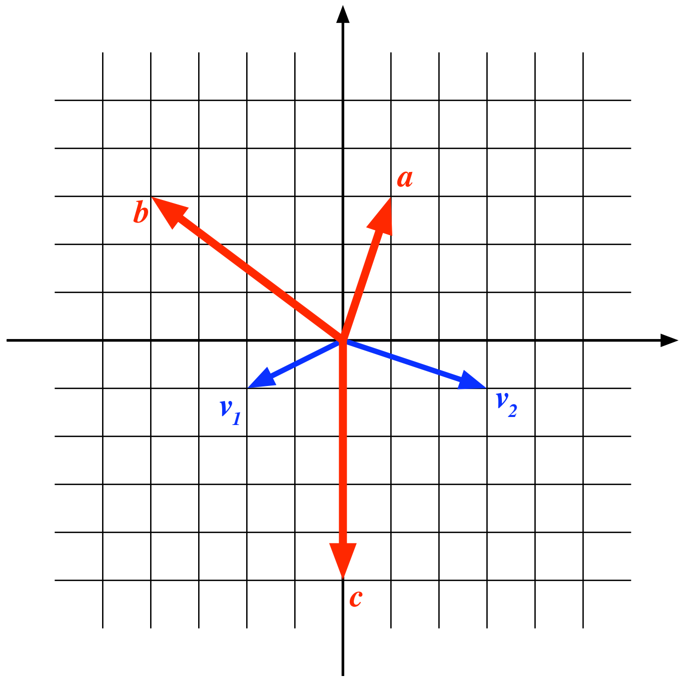
$$\vec{v} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} \qquad \vec{u}_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \qquad \vec{u}_2 = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

Problem 4. (6 points)
Find all values of h, such that the vectors

$$\begin{pmatrix} 1 \\ 1 \\ 2 \\ 3 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ h \\ 3 \\ 5 \end{pmatrix} \qquad \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$$

are linearly independent.

Problem 5. (5 points)
Refering to the sketch, write the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ as linear combinations of $\vec{v}_1$ and $\vec{v}_2$.



Problem 6. (6 points)
For each transformation decide whether or not it is linear. If it is linear, find its matrix.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ y-x \\ x \end{pmatrix}$.

(b) $S : \mathbb{R}^4 \to \mathbb{R}^4$, which multiplies every vector by the scalar 5.

(c) $U : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $U \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y+1 \\ x-y-1 \end{pmatrix}$

Problem 7. (6 points)
Consider the linear transformations S, and T, both with domain $\mathbb{R}^2$ and codomain $\mathbb{R}^2$.

(a) The transformation S first rotates every vector by an angle of $\pi/2$ counterclockwise about the origin, and then reflects the resulting vector across the x-axis. Find the matrix of S.

$[S] =$

(b) The transformation T first reflects every vector across the x-axis, and then rotates the resulting vector counterclockwise about the origin by an angle of $\pi/2$. Find the matrix of T.

$[T] =$

(c) Describe the transformation S as a single geometric operation. Do the same for T.