Practice Midterm Exam I

No books. No notes. No calculators. No electronic devices of any kind.

Problem 1. (0 points)
What conditions do x, y, z, u, w have to satisfy for the matrix

$$A = \begin{pmatrix} 0 & 1 & y & u & w \\ 0 & 0 & 0 & z & 0 \\ 0 & x & 0 & 0 & 1 \end{pmatrix}$$

to be in reduced row echelon form?

Problem 2. (0 points)
List all matrices in reduced row echelon form, whose first row is $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$.

Problem 3. (0 points)
Find out for which values of k the vector $\vec{v}$ is a linear combination of $\vec{u}_1$ and $\vec{u}_2$

$$\vec{v} = \begin{pmatrix} k \\ 2 \\ k \end{pmatrix} \qquad \vec{u}_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \qquad \vec{u}_2 = \begin{pmatrix} 13 \\ 1 \\ 8 \end{pmatrix}$$

Problem 4. (0 points)
Determine whether the vectors

$$\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

are linearly dependent or linearly independent. Justify your answer.

Problem 5. (0 points)
Find an equation defining the plane through the origin in $\mathbb{R}^3$, which is spanned by the two vectors

$$\vec{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad \vec{u}_2 = \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}$$

Problem 6. (0 points)
Find two vectors in $\mathbb{R}^3$, which span the plane defined by the equation

$$2x - 3y + 6z = 0$$

Problem 7. (0 points)
Decide whether or not $\vec{v} = \left(\begin{smallmatrix} 1 \\ 2 \\ 3 \end{smallmatrix}\right)$ is contained in $W = \operatorname{span}\{\left(\begin{smallmatrix} 1 \\ 0 \\ -1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 \\ -1 \\ 1 \end{smallmatrix}\right)\}$.

Problem 8. (6 points)
Write down the general solution in parametric vector form. Your answer will depend on k.

$$\begin{array}{rcrcrcrcl} 2x_1 & - & 4x_2 & + & x_3 & + & kx_4 & = & 0 \\ x_1 & - & 2x_2 & + & 2x_3 & + & kx_4 & = & 0 \\ x_1 & - & 2x_2 & + & x_3 & + & 2kx_4 & = & 0 \\ x_1 & - & 2x_2 & + & x_3 & + & kx_4 & = & 0 \end{array}$$

Problem 9. (0 points)
For which values of k are the vectors $\left(\begin{smallmatrix} 1 \\ k \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 2 \\ 5k-1 \end{smallmatrix}\right)$ linearly independent?

Problem 10. (0 points)
Linear transformations:

(a) Find the matrix of the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^2$, defined by the formula
$$T(a, b, c, d) = (a - b, a + c - d)\,.$$

(b) Find the matrix of the linear transformation $R : \mathbb{R}^2 \to \mathbb{R}^2$, which rotates every vector by an angle of π about the origin.

(c) Find the matrix of the linear transformation $S : \mathbb{R}^3 \to \mathbb{R}^3$, which rotates every vector by an angle of $\frac{\pi}{2}$ about the x_2-axis. (The rotation is counterclockwise, if you view the (x_1, x_3)-plane from the positive x_2-axis.)

Problem 11. (0 points)
Consider the two matrices

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}.$$

Both A and B define matrix transformations.

(a) The domain of A is ______, the codomain of A is ______.

(b) Is A onto? Show your work.

(c) The domain of B is ______, the codomain of B is ______.

(d) Is B one-to-one? Show your work.

Problem 12. (0 points)
Consider the matrices

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Find the matrix of the linear transformation which first multiplies by A and then by B.

Problem 13. (0 points)
Find the matrix of the linear transformation which reflects across the line $x+2y = 0$.

Problem 14. (0 points)
Describe geometrically the matrix transformation given by $\left(\begin{smallmatrix} 3 & -4 \\ 4 & -3 \end{smallmatrix}\right)$.

Problem 15. (0 points)
Describe geometrically the range of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$, whose matrix is

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$