Quiz 3 - Math221 - Sec 203 Jan 25, 2017

Name: _____ Student number: _____

No books. No notes. No calculator. No electronic device of any kind.

1. (*4 points*) Determine for what value or values of h the following vectors are linearly independent? Show your work.

$a_1 x_1 + a_2 x_2 + a_3 x_3 = 0$

Only trivial solution

$$a_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \qquad a_2 = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix} \qquad a_3 = \begin{bmatrix} 4 \\ 10 \\ h \end{bmatrix}$$

$$\begin{pmatrix} 1 & 2 & 4 \\ 1 & 4 & 10 \\ 2 & 4 & h \end{pmatrix} \underset{R_3 - 2R_1}{\overset{R_2 - R_1}{\sim}} \begin{pmatrix} \boxed{1} & 2 & 4 \\ 0 & \boxed{2} & 6 \\ 0 & 0 & \boxed{h-8} \end{pmatrix} \text{REF} \qquad (1\text{ pt})$$

in order to have only trivial solut. $\Rightarrow$ No free variable $\Rightarrow$ one pivot @ each column $\Rightarrow h - 8 \neq 0$

(2 pts) $\boxed{h \neq 8}$

2. (*4 points*) Find the standard matrix for a linear transformation $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ with the following features: it first reflects the points through the line $x_1 = x_2$. Next, it expands every point horizontally with a factor of $c = 2$. Show your work.

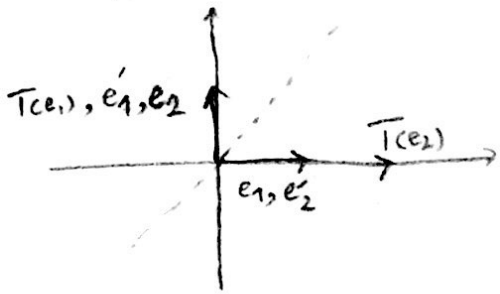
$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{\text{reflection}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\text{expan.}} \begin{bmatrix} 2(0) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = T(e_1)$$

$$e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\text{reflection}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{\text{expan.}} \begin{bmatrix} 2(1) \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = T(e_2) \qquad (2\text{ pts.})$$

$$A = [T(e_1)\ T(e_2)] = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \qquad 2\text{ (pts.)}$$

figure is just for visualizing the solution and is not mandatory!

reflection & expansion



3. (*4 points*) Each of the following standard matrices correspond to a linear transformation of $T(x) : \mathbb{R}^n \mapsto \mathbb{R}^m$ (recall the matrix form $Ax = b$). For the linear transformation corresponding to each matrix answer these two questions: Does the linear transformation map $\mathbb{R}^n$ onto $\mathbb{R}^m$? Is the linear transformation one-to-one? No need to explain why.

- $T(x) : \mathbb{R}^3 \mapsto \mathbb{R}^2$, $\quad A = \begin{pmatrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{1} & 3 \end{pmatrix}$

  onto $\mathbb{R}^2$: Yes 1pt

  one-to-one : No 1pt

- $T(x) : \mathbb{R}^2 \mapsto \mathbb{R}^3$, $\quad A = \begin{pmatrix} \boxed{1} & 2 \\ 0 & \boxed{1} \\ 0 & 0 \end{pmatrix}$

  onto $\mathbb{R}^3$: No 1pt

  one-to-one : Yes 1pt