Quiz 4 - Math221 - Sec 203 Feb 7, 2017

Name: ______________________ Student number: ______________

No books. No notes. No calculator. No electronic device of any kind.

1. (*2 points*) Show if AB is invertible, then B is also invertible.

AB is invertible : $(AB)^{-1} = W$

$W(AB) = I \Rightarrow (WA)B = I \Rightarrow B$ is invertible

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2. (*4 points*) An argument in form of "if <statement 1> then <statement 2>" is an implication. It means that if <statement 1> is true then <statement 2> is also always true. Determine if the following implications are true or false. If it is true, justify your answer. If it is false, find a counterexample where <statement 1> is true but <statement 2> is false. For all implications, A is a $n \times n$ matrix.

1. If the columns of A span $\mathbb{R}^n$, then the columns are linearly independent. T
2. If the equation $Ax = 0$ has only the trivial solution, then the A is row equivalent to the identity matrix. T
3. If the equation $Ax = 0$ is consistent, then it has a unique solution. F
4. If A is the standard matrix corresponding to a linear transformation which is one-to-one, then A is invertible. T

(1) Cols of A span $\mathbb{R}^n \Rightarrow$ one pivot @ each row $\overset{n\times n}{\Rightarrow}$ one pivot @ each col.

$\Rightarrow$ cols are linearly independant.

(2) $Ax=0$ only trivial solut. $\Rightarrow$ cols are linearly independant $\Rightarrow$ one pivot @ each col $\overset{n\times n}{\Longrightarrow}$ one pivot @ each row $\Rightarrow$ A is invertible

(3) counter example: $n=3$, $r=2$ (nb. pivot) $\Rightarrow$ 1 free variable $\Rightarrow$ not unique solut.
and allways consistant

(4) A is 1-to-1 $\Rightarrow$ one pivot each col. $\Rightarrow n$ pivots $\Rightarrow$ A is invertible.