Quiz 6 - Math221 - Sec 203 Mar 1, 2017

Name: ______________________ Student number: ______________________

No books. No notes. No calculator. No electronic device of any kind.

1. (*12 points*) Find the determinant of the following matrices, A, B and C. You can use either the forward elimination method or the co-factor expansion method. In (a) and (b), your solution should be a number, while in (c) your solution should depend on α and β.

(a) $A = \begin{pmatrix} 3 & 5 & -2 & 6 \\ 2 & 4 & 1 & 5 \\ 1 & 2 & -1 & 1 \\ 3 & 7 & 1 & 3 \end{pmatrix}$

Row reduction:

$$\det A = -\det\begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 5 & -2 & 6 \\ 3 & 7 & 1 & 3 \end{pmatrix} = -\det\begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & -1 & 1 & 3 \\ 0 & 1 & 4 & 0 \end{pmatrix}\begin{matrix} R_2-2R_1 \\ R_3-3R_1 \\ R_4-3R_1 \end{matrix}$$

$$= \det\begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 3 & 3 \end{pmatrix} R_4-\tfrac{3}{5}R_3 = \det\begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 6/5 \end{pmatrix} = (1)(1)(5)\left(\tfrac{6}{5}\right) = \underline{6}$$

(b) $B = 2A^{-1}A^tA^{-1}$

$$\det(B) = \det(2A^{-1}A^tA^{-1}) = 2^4\frac{\det(A)}{\det(A)\det(A)} = 16\cdot\frac{1}{6} = 8/3$$

(c) $C = \begin{pmatrix} 5 & \beta+1 & 0 & 23 \\ 2 & 0 & 0 & 0 \\ 100 & \beta & \alpha & 20 \\ 0 & 0 & 0 & 5 \end{pmatrix}$

cofactor expansion on 2nd row:

$$-2 \underbrace{\begin{vmatrix} \beta+1 & 0 & 23 \\ \beta & \alpha & 20 \\ 0 & 0 & 5 \end{vmatrix}}_{\text{expand } 3^{rd} \text{ row}} = -2\left(5 \begin{vmatrix} \beta+1 & 0 \\ \beta & \alpha \end{vmatrix}\right) = -10(\beta+1)(\alpha)$$

2. (*2 points*) For what values of α and β in the Problem 1(c), C is **not invertible.**

$\det(C) = 0$

either $\alpha = 0$ or $\beta = -1$