Quiz 7 - Math221 - Sec 203 Mar 15, 2017

Name: ______________________ Student number: ______________

No books. No notes. No calculator. No electronic device of any kind.

1. (*12 points*) For the following discrete dynamical system

$$x_{n+1} = \frac{3}{2}x_n - \frac{1}{2}y_n$$

$$y_{n+1} = -x_n + y_n$$

(a) Find the general solution.

(b) If $x_0 = 1$ and $y_0 = 3$, find an explicit formula for $\begin{bmatrix} x_n \\ y_n \end{bmatrix}$.

(c) Sketch the phase portrait

$f_{n+1} = A f_n \quad A = \begin{pmatrix} 3/2 & -1/2 \\ -1 & 1 \end{pmatrix}$

$(\lambda - 3/2)(\lambda - 1) - 1/2 = 0 \Rightarrow \lambda^2 - \frac{5}{2}\lambda + 1 = 0 \quad \frac{5 \pm \sqrt{25-16}}{4} \begin{cases} \lambda_1 = 2 \\ \lambda_2 = 1/2 \end{cases}$

$\underline{\lambda = 2}$ $(A - 2I_2)x = 0$

$\begin{pmatrix} -1/2 & -1/2 \\ -1 & -1 \end{pmatrix} \sim \begin{pmatrix} \boxed{1} & 1 \\ 0 & 0 \end{pmatrix}$ $x_1 = -x_2$, x_2 free $\quad x = r\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $r \in \mathbb{R}$

$\lambda = 1/2$ $(A - \frac{1}{2}I_2)x = 0$

$\begin{pmatrix} 1 & -1/2 \\ -1 & 1/2 \end{pmatrix} \sim \begin{pmatrix} \boxed{1} & -1/2 \\ 0 & 0 \end{pmatrix}$ $x_1 = x_2/2$, x_2 free $\quad x = r\begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$ $r \in \mathbb{R}$

a) $f_n = c_1(2)^n\begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2(1/2)^n\begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 \\ 3 \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$

$$\left(\begin{array}{cc|c} -1 & 1/2 & 1 \\ 1 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 3/2 & 4 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 1/3 \\ 0 & 1 & 8/3 \end{array}\right)$$

$c_1 = 1/3 \qquad c_2 = 8/3$

$$f_n = \frac{1}{3}(2)^n \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \frac{8}{3}(1/2)^n \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$$

c)