13.5 POWER OF A POWER PROPERTY

To solve an equation with more than one exponent associated with only one base, such as $(5^4)^3$, we can use the exponent on the outside of the brackets to tell us how many times to write the expression inside the brackets. $\therefore (5^4)^3 = 5^4 \times 5^4 \times 5^4 = 5^{12}$

But if we use the **power of a power property,** we can exponentially solve this question by multiplying as shown in the examples below.

EXAMPLES: Use the power of a power property to solve each and leave your answer in exponential form.

1. $(12^4)^5 = 12^{4 \times 5} = 12^{20}$

2. $[(8^{-3})^7]^2 = 8^{-3 \cdot 7 \cdot 2} = 8^{-42}$

3. $(5^2 \cdot 6^3)^4 = 5^{2 \cdot 4} \cdot 6^{3 \cdot 4}$
$= 5^8 \cdot 6^{12}$

4. $(3^4 a^2 b^3)^5 = 3^{4 \cdot 5} a^{2 \cdot 5} b^{3 \cdot 5}$
$= 3^{20} a^{10} b^{15}$

A. Use the power of a power property and write each out in simplest exponential notation.

1. $(2^4)^3$ 2^{12}
2. $(3^2)^6$ 3^{12}
3. $(5^3)^4$ 5^{12}
4. $(6^2)^{-4}$ 6^{-8}
5. $(3^2)^{-5}$ 3^{-10}
6. $(8^{-2})^6$ 8^{-12}
7. $(3^{-8})^3$ 3^{-24}
8. $(2^{-5})^{-2}$ 2^{10}
9. $(2^{-3})^{-4}$ 2^{12}
10. $(7^{-4})^{-3}$ 7^{12}
11. $(6^{-4})^5$ 6^{-20}
12. $(5^0)^{-3}$ 5^0
13. $(4^{-7})^4$ 4^{-28}
14. $(7^8)^0$ 7^0
15. $(15^3)^{-7}$ 15^{-21}
16. $(3^0)^0$ 3^0
17. $(y^3)^2$ y^6
18. $(m^2)^4$ m^8
19. $(x^5)^7$ x^{35}
20. $(z^4)^5$ z^{20}
21. $(y^3)^2$ y^6
22. $(z^{-3})^4$ z^{-12}
23. $(z^{-5})^6$ z^{-30}
24. $(y^{-6})^{-2}$ y^{12}
25. $(a^{-7})^{-2}$ a^{14}
26. $(x^0)^7$ x^0
27. $(z^{-8})^0$ z^0
28. $(y^0)^0$ y^0
29. $(m^{-8})^{-7}$ m^{56}
30. $(y^5)^3$ y^{15}
31. $(7^2)^2$ 7^4
32. $(3^2)^2$ 3^4
33. $(5^3)^4$ 5^{12}
34. $(6^2)^3$ 6^6
35. $(m^{-3})^3$ m^{-9}
36. $(5^{-4})^2$ 5^{-8}
37. $(6^{-4})^3$ 6^{-12}
38. $(8^3)^5$ 8^{15}
39. $(3^{16})^{23}$ 3^{368}
40. $(45^8)^{-34}$ 45^{-272}
41. $(67^{20})^{-20}$ 67^{-400}
42. $(67^{67})^{67}$ 67^{4489}
43. $(12^{13})^{13}$ 12^{169}
44. $(5^{65})^0$ 5^0
45. $[(x^7)^5]^2$ x^{70}

B. Use the power of a power property to solve each of the following.

1. $(3^2)^3$ 3^6
2. $(6^5)^3$ 6^{15}
3. $(7^6)^4$ 7^{24}
4. $(x^3)^2$ x^6
5. $(-3)^2$ $(-3)^2$
6. $(3^{-2})^2$ 3^{-4}
7. $(4^{-2})^{-3}$ 4^6
8. $(3x^2)^2$ 3^2x^4
9. $(ab)^5$ a^5b^5
10. $(3 \cdot 4)^7$ $3^7 4^7$
11. $(a^2 b^3)^2$ $a^4 b^6$
12. $(x^2 y^3)^5$ $x^{10} y^{15}$
13. $(a^4 b^{-5})^6$ $a^{24} b^{-30}$
14. $(a^2/b^3)^4$ $\frac{a^8}{b^{12}}$
15. $(5x^2)^4$ $5^4 x^8$
16. $\left\{\frac{x}{3}\right\}^5$ $\frac{x^5}{3^5}$
17. $(4a^2b)^{-5}$ $4^{-5} a^{-10} b^{-5}$
18. $\left\{\frac{3}{x}\right\}^{-3}$ $\frac{3^{-3}}{x^{-3}}$
19. $(-3y^3)^7$ $(-3)^7 y^{21}$
20. $\left\{\frac{5x}{3y}\right\}^5$ $\frac{5^5 x^5}{3^5 y^5}$
21. $[(5^2)^3]^0$ 5^0
22. $(y^4)^{-3}$ y^{-12}
23. $(x^2 y^8)^{-6}$ $x^{-12} y^{-48}$
24. $(x^2 y^2 z^2)^3$ $x^6 y^6 z^6$
25. $(y^{-3} z^{-1})^5$ $y^{-15} \cdot z^{-5}$
26. $(8^2 y^3 z^5)^2$ $8^4 y^6 z^{10}$
27. $(r^7 s^5 t^4)^3$ $r^{21} s^{15} t^{12}$
28. $\left\{\frac{x^2}{4}\right\}^2$ $\frac{x^4}{4^2}$
29. $(a^2 b^2)^3$ $a^6 b^6$
30. $(3x^3)^7$ $3^7 x^{21}$
31. $(5y^2 z^3)^7$ $5^7 y^{14} z^{21}$
32. $(xy)^2$ $x^2 y^2$
33. $(3^5)^7$ 3^{35}
34. $[(ab)^2]^3$ $a^6 b^6$
35. $(3 a^2 b^{-3})^4$ $3^4 a^8 b^{-12}$
36. $[(1^2)^3]^5$ 1^{30}
37. $(m^2 n^2)^3$ $m^6 n^6$
38. $[(x^2)^4]^3$ x^{24}
39. $[(a^2 b^3)^{-4}]^{-3}$ $a^{24} b^{36}$
40. $[(7^2)^3]^4$ 7^{24}
41. $[(a^2 b)^4]^3$ $a^{24} b^{12}$
42. $\left\{\frac{3a^2 b^3}{c^4}\right\}^7$ $\frac{3^7 a^{14} b^{21}}{c^{28}}$

C. Use all the rules for exponents you have already taken and write each answer in simplest exponential form.

1. $5^3 \times 5^2 \times 5^5$ 5^{10}
2. $(5^3)^2$ 5^6
3. $(4^{-4})^2$ 4^{-8}
4. $5^{-2} \cdot 5^{-5}$ 5^{-7}
5. $(x^2)^3 (x^5)^3$ x^{21}
6. $z^{-8} \div z^{-6}$ z^{-2}
7. $(0.9)^6 (0.9)^7 (0.9)$ 0.9^{14}
8. $(x^5 y^{-3}) \div (x^{-3} y^7)$ $x^{8} y^{-10}$
9. $(2 \cdot 3 \cdot x)^6$ $2^6 \cdot 3^6 \cdot x^6$
10. $(9^2)^4 (9^2)^5$ 9^{18}
11. $(xyz)^3 (xy^2)^2$ $x^5 y^7 z^3$
12. $(2xy)^5$ $2^5 x^5 y^5$
13. $(x^4 y^5) \div (x y^{-4})$ $x^3 y^9$
14. $x^{-2} \div x^{-3}$ x^1
15. $(y^2)^3 (y^3)^5 (y^5)^{-3}$ y^6
16. $(y^7) \cdot (y^8) \div (y^3)$ y^{12}
17. $(a^4)^2 (a^5)^3$ a^{23}
18. $a^4 \cdot a^2 \div a^3$ a^3
19. $[(x^0)^{10}]^{21}$ x^0
20. $x^6 y^7 x^8 y^9$ $x^{14} y^{16}$
21. $(a^2 b)^7$ $a^{14} b^7$
22. $\{[(5^2)^3]^4\}^5$ 5^{120}
23. $\left\{\frac{a^2 b^3 c^6}{a^3 b^6 c^8}\right\}^7$ $a^{-7} b^{-21} c^{-14}$
24. $(5 a^3 b^{-3} c^5)^6$ $5^6 a^{18} b^{-18} c^{30}$