

Part-time and full-time work behaviour of married women: a model with a doubly truncated dependent variable

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Abstract. Most micro-labour supply models presented in the literature embody the implicit assumption that the coefficients of the explanatory variables are stable over the range of variation for annual hours of work. In this paper we discuss reasons why this may not be true. A two-stage consistent estimation method that eliminates the sample selection bias problem for samples censored at two limits and is an extension of a consistent two-stage estimation method presented by James Heckman for the case of samples censored at a single limit is then used to explore this question empirically.

Le comportement des femmes mariées travaillant à temps plein et à temps partiel: un modèle où la variable dépendante est tronquée aux deux bouts. La plupart des micro-modèles d'offre de travail qu'on trouve dans la littérature spécialisée contiennent le postulat implicite que les coefficients des variables indépendantes sont stables sur tout l'éventail du nombre des heures de travail au cours d'une année. Les auteurs examinent les raisons pour lesquelles ce n'est pas le cas. Ce travail empirique utilise une méthode de calibration consistante en deux étapes qui élimine le problème du biais dans l'échantillonnage pour des échantillons tronqués aux deux bouts. Il s'agit d'une extension de la méthode mise au point par James Heckman qui ne tronquait la gamme d'échantillons qu'à un seul bout.

INTRODUCTION

In this paper we examine simplifications which are typically made in order to obtain an empirically tractable model of the labour force behaviour of married women, and we discuss the reasons that some of the parameters of the simplified empirical model may take on different values over the range of variation in the selected measure of labour supply. If parameter instability of this sort exists, then if a sample of working wives is split into two groups depending on the amount of labour supplied by each wife and a model of labour force behaviour is estimated using data for each of these groups, we would expect to find statistically significant differences between the

This research was supported by Social Sciences and Humanities Research Council of Canada Research Grant 410-77-0339-R1. The authors acknowledge helpful comments made on earlier versions of the paper by Professors Marianne A. Ferber, Victor R Fuchs, Daniel S Hamermesh, Guy H. Orcutt, Hiroki Tsurumi, Alan D. Woodland, and the participants in a seminar given at the University of Michigan. Helpful comments by Glenn MacDonald on a later version of the paper are also gratefully acknowledged.

Canadian Journal of Economics / Revue canadienne d'Économique, XVI, no. 2
May / mai 1983. Printed in Canada / Imprimé au Canada

0008-4085 / 83 / 0000-0229 \$01.50 ©1983 Canadian Economics Association

resulting sets of estimates for the parameters of the model. If parameter instability of this sort is found to exist, the next question is does it matter? Do the signs of any of the coefficients change over the range of variation in the measure of labour supply? Do these results shed any light on behavioural hypotheses of interest? Are there any policy implications of these results? Would it be important for us in some way to take account of this parameter instability in estimating equations to be used in predicting the labour supply and earnings of individuals?

A two-stage consistent estimation method is used to obtain separate sets of estimates of the parameters of a model of labour force behaviour for U.S. and Canadian wives working less than 1,400 hours and for those working at least 1,400 hours. The estimation method employed explicitly accounts for the sample selection bias problem resulting from the censoring of a sample at two limits, and is a straightforward extension of a consistent two-stage estimation method presented by James Heckman (1976, 1979) for the case of samples censored at a single limit. The piece-wise approximation that can be obtained using this estimation method is in the same spirit as the approximation obtained by substituting several dummy variables (used either to shift the constant or one or more slopes in an equation) for a continuous explanatory variable such as age or education when the form of the functional relationship between the dependent variable and the explanatory variable is unknown.¹ That is, the approximation will be discontinuous at the end-points. Such an approximation will not necessarily satisfy a perceived need for a fully specified behavioural model, including the functional forms of all behavioural interrelationships. Such an approximation, however, may at least represent a valuable step along the road to further understanding.² For convenience we shall refer to wives working less than 1,400 hours in a year as part-time workers, and to those working at least 1,400 hours as full-time workers.

The decision to split working wives into two groups depending on whether they worked more or less than 1,400 hours in the year is clearly arbitrary. The figure of 1,400 hours was chosen simply because it results in fairly equal numbers of wives in our so-called part-time and full-time hours of work categories. Since virtually all the published studies of the labour force behaviour of women, or married women, implicitly embody our null hypothesis that the coefficients of these models are stable over the range of variation for the chosen measure of labour supply, it is appropriate to begin in this study by simply asking whether this null hypothesis can be rejected when wives are split into low and high hours of work categories.

We are particularly interested in determining whether there are any systematic

1 A dummy variable slope shifter cannot be used to check for differences in coefficient values associated with different numbers of hours of work, however, since hours of work is an endogenous variable in our model.

2 Wales and Woodland (1978, 38) note, for instance, that 'A nonlinear model causes no real problems for the maximum likelihood procedures either conceptually or computationally.' Maximum likelihood procedures cannot be applied, however, until the functional nature of the relevant non-linearities has been fully determined. Moreover, translating even well specified non-linear relationships, such as the non-linear relationships between before- and after-tax earned income as defined by the relevant tax tables, into functional forms may sometimes be very difficult.

differences in the uncompensated wage elasticities of hours of work for wives who work more versus those who work less than 1,400 hours in the year. Most researchers have found the uncompensated wage elasticity of hours of work to be positive for working wives, but negative for men. It is sometimes suggested, however, that the wage elasticity for wives who work full time may be negative, just as it is for men who work predominantly full-time. Empirical verification of this point has been lacking. Yet if it were true that this elasticity is positive for women working small numbers of hours, but negative for those working full time, this would raise the possibility that some of the conflicting results presented in the literature may be due to the fact that the ratio of part-time to full-time working wives is different in different data bases and in the differing samples which different researchers extract from these data bases.³

A variety of other topics are also investigated in this paper. An improved index is introduced to account for the local demand for the labour of women of different types. Problems of multicollinearity between the selection bias term introduced into our regression equations and the other explanatory variables are discussed, and an unconventional instrumental variable estimator proposed by Durbin (1954) is used for the wage variable in our hours equation in an effort to cope with some of these problems. Difficulties are reported with respect to the theoretically suggested correction for heteroscedasticity for our wage and hours equations. Also we explore the question of whether it would matter if we took account of regional price differences in our empirical specifications.

A MODEL OF THE LABOUR FORCE BEHAVIOUR
OF MARRIED WOMEN

The basic model adopted in this paper is developed more fully in Nakamura and Nakamura (1981). It is assumed that a family maximizes a twice differentiable quasiconcave conditional utility function $U(x, T - h; E_H T, Z^*)$ subject to the time constraint $0 \leq h < T$ and the one period budget constraint

$$px = E_H T + w \int_0^h (1 - \tau x_s) ds, \tag{1}$$

where x is a Hicksian composite good with unit price p , h represents the annual hours of work of the wife at wage w , $E_H T$ is the husband's earned income plus family asset income net of the income taxes which would be paid at zero hours of work for the wife, Z^* is a vector of predetermined constraints, T is the wife's total time, and τx_s is the marginal tax rate on the wife's earnings at s hours of work. The wife's marginal

3 In Nakamura, Nakamura, and Cullen (1979) and in Nakamura and Nakamura (1981) the uncompensated wage elasticities of hours of work for working married women in Canada, and in the United States and Canada, respectively, are found to be negative and of roughly the same magnitude as those reported by other researchers for men. The results shown are for several different age groups of married women.

net offered wage at h hours of work is given by

$$w_h^n = w(1 - \tau x_h). \tag{2}$$

Maximizing the Lagrangian

$$L = U(x, T - h; E_H T, Z^*) + \lambda \left\{ E_H T + w \int_0^h (1 - \tau x_s) ds - px \right\} + \gamma h \tag{3}$$

with respect to x and h , where λ and γ (≥ 0) are Lagrange multipliers, leads to the conditions⁴ that a wife will work only if

$$w_h^n \geq w_h^* \quad \text{at} \quad h = 0 \tag{4}$$

and that wives who work will adjust their hours of work such that

$$w_h^n = w_h^*, \tag{5}$$

with the shadow price of the wife's time (her asking wage) at h hours of work defined by

$$w_h^* = U_l / \lambda, \tag{6}$$

where $\lambda = U_x / p$ and $l = T - h$. Linearizing the right hand side of (1) around h , yields

$$px = E_H T + w_h^n h. \tag{7}$$

Using this approximate budget constraint, (6) and $\lambda = U_x / p$, the wife's asking wage, w_h^* , may be represented as a function of h , $E_H T$, $w_h^n h$ and Z^* when $h > 0$; and a function of $E_H T$ and Z^* when $h = 0$. Making use of (7), taking the log of both sides of (6) and linearizing this expression around Z^* , $E_H T$, $\ln w_h^n$ and h yields

$$\ln w_h^* = \begin{cases} \beta_0 + Z^* \beta_1 + \beta_2 E_H T + \beta_3 \ln w_h^n + \beta_4 h + U^* & \text{if } h > 0 \\ \beta_0 + Z^* \beta_1 + \beta_2 E_H T + U^* & \text{if } h = 0, \end{cases} \tag{8}$$

where U^* denotes the disturbance term. It is also assumed in that variations in the log of the wife's offered wage w are explained by

$$\ln w = \alpha_0 + Z \alpha_1 + R \alpha_2 + u, \tag{9}$$

where Z and R are, respectively, vectors of personal characteristics and regional macro-economic variables and u denotes the disturbance term.

Notice that in deriving the simplified empirical model given by (8) and (9), interactions between h and w_h^n , the after-tax marginal wage rate, are ignored in the approximation to the budget constraint given in (7). Interactions between h and w_h^n are also ignored in (8) in linearizing the asking wage function. In (9) it is simply assumed that $\ln w$ does not depend on how many hours a wife works. Finally, in both (8) and (9) the constant terms are assumed to be the same for all wives. Suppose,

4 Necessary conditions are $U_x - \lambda p = 0$, $-U_l + \lambda \{w(1 - \tau x_h)\} + \gamma = 0$ and $\gamma h = 0$, where $U_l = \partial U(x, l; E_H T, Z^*) / \partial l$ and $U_x = \partial U(x, l; E_H T, Z^*) / \partial x$. These, in turn, imply $w(1 - \tau x_h) + (\gamma / \lambda) = U_l / \gamma$ or $w_h^n + (\gamma / \lambda) = w_h^*$. Since $\gamma = 0$ if $h > 0$ (and $\gamma \geq 0$ if $h = 0$), we get (4) and (5).

however, that there are fixed or persistent unobservable factors that have important impacts on the labour force behaviour of individual wives. Suppose, furthermore, that the mean values of these unobservable fixed or persistent effects are quite different for part-time versus full-time working wives. Constraining the intercepts of (8) and (9) may lead to fixed effects coefficient biases, and the equation error terms may have non-zero means and fail to obey normal distributions even though U^* and u are approximately normally distributed. These simplifications appear, either explicitly or implicitly, in a number of other empirical papers on the labour force behaviour of married women.⁵ Moreover in studies such as Heckman's (1974, 1976) where the impact of income taxes is ignored, the dependence of TX_h on h in (2) is also ignored. Problems resulting from fixed or persistent effects are of special concern in a Canadian context. There are no good panel data for individuals in Canada. Thus there is no possibility in Canadian studies of using econometric methods that make use of panel data to allow for or estimate individual fixed effects terms or autoregressive error structures. Any one of these simplifications that we have listed could potentially result in parameter instability over the range of values for h .

In the case of couples in the United States who file joint tax returns, the first dollar of a wife's earnings is taxed essentially at the marginal rate that would apply to an additional dollar earned by the husband. In Canada, on the other hand, working husbands and wives must file separate tax returns. Defining the marginal retention rate at h hours of work as

$$RET_h = 1 - TX_h, \tag{10}$$

for a separate return at $h = 0$ we have

$$\ln (RET_0) = 0. \tag{11}$$

For a joint return, linearizing $\ln (RET)$ evaluated at $h = 0$ around E_{HT} yields

$$\ln (RET_0) = \eta_0 + \eta_1(E_{HT}) + u' \tag{12}$$

where η_0 and η_1 are parameters and u' is a random disturbance term which is assumed to be normally distributed with mean 0 and constant variance. For some given positive number of annual hours of work, h_G , linearizing $\ln (RET)$ evaluated at $h = h_G$ around E_{HT} , Z , R , Z^* , and h_G yields

$$\ln (RET_{h_G}) = \eta_0 + \eta_1(E_{HT}) + Z\eta_2 + R\eta_3 + Z^*\eta_4 + \eta_5 h_G + u'' \tag{13}$$

for a joint return, while for a separate return evaluated at $h = h_G$ we have

$$\ln (RET_{h_G}) = \eta_0 + Z\eta_2 + R\eta_3 + Z^*\eta_4 + \eta_5 h_G + u''', \tag{14}$$

where η_2 through η_5 are parameters, and u'' and u''' are random disturbance terms assumed to be normally distributed with mean 0 and constant variances. Thus from

5 See, for instance, Heckman (1974, 1976). In Rosen (1976) the offered wage is allowed to depend on the hours of work, but a linearization is employed in accounting for income taxes and the hours of work function can be seen to result from implicit linearizations of the asking wage function of the sort discussed above.

(2), (4) and (8) through (14) we see that the probability a wife will work more than h_G hours may be expressed as

$$P(h > h_G) = P(\ln w_h^n - \ln w_h^*|_{h=h_G} > 0) \tag{15}$$

$$= (1/(2\pi)^{\frac{1}{2}}) \int_{-\infty}^{\phi_{h_G}} e^{-(t^2/2)} dt,$$

where⁶

$$\phi_0 = (1/\sigma_0)[(\alpha_0 - \beta_0) + Z\alpha_1 + R\alpha_2 - Z^*\beta_1 - \beta_2 E_{HT}] \tag{16}$$

for a separate return and $h_G = 0$ with σ_0^2 denoting the variance of the random term $U^* - u$,

$$\phi_0 = (1/\sigma_0')[(\alpha_0 + \eta_0 - \beta_0) + Z\alpha_1 + R\alpha_2 - Z^*\beta_1 + (\eta_1 - \beta_2)E_{HT}] \tag{17}$$

for a joint return and $h_G = 0$ with $(\sigma_0')^2$ denoting the variance of the random term $U^* - (u + u')$,

$$\phi_{h_G} = (1/\sigma_{h_G})[(1 - \beta_3)(\alpha_0 + \eta_0) - \beta_0 + ((1 - \beta_3)\eta_5 - \beta_4)h_G \tag{18}$$

$$+ Z(1 - \beta_3)(\alpha_1 + \eta_2) + R(1 - \beta_3)(\alpha_2 + \eta_3)$$

$$+ Z^*((1 - \beta_3)\eta_4 - \beta_1) - \beta_2 E_{HT}]$$

for a separate return and $h_G > 0$ with $\sigma_{h_G}^2$ denoting the variance of the random term $U^* - (1 - \beta_3)(u + u'')$, and

$$\phi_{h_G} = (1/\sigma_{h_G}')[(1 - \beta_3)(\alpha_0 + \eta_0) - \beta_0 + ((1 - \beta_3)\eta_5 - \beta_4)h_G \tag{19}$$

$$+ Z(1 - \beta_3)(\alpha_1 + \eta_2) + R(1 - \beta_3)(\alpha_2 + \eta_3)$$

$$+ Z^*((1 - \beta_3)\eta_4 - \beta_1) + ((1 - \beta_3)\eta_1 - \beta_2)E_{HT}],$$

for a joint return and $h_G > 0$ with $(\sigma_{h_G}')^2$ denoting the variance of the random term $U^* - (1 - \beta_3)(u + u'')$.

From (5), (2), and (8) we see also that for wives who do work

$$h = (1/\beta_4)[- \beta_0 + (1 - \beta_3) \ln w_h^n - Z^*\beta_1 - \beta_2 E_{HT}] - (1/\beta_4)U^*. \tag{20}$$

In this study we are interested in obtaining separate sets of estimates of the parameters of (9), our equation for the log of the offered wage, and of (20), our equation for the wife's annual hours of work, for wives working less than 1,400 hours and for those working at least 1,400 hours.

Equations (9) and (20) cannot be estimated directly, because of the selection bias problem and because $\ln w_h^n$ and U^* are correlated in (20). It can be shown, however, that the reduced form of (20) may be written as⁷

$$h = (1/\beta_4)[- \beta_0 + (1 - \beta_3) \overline{\ln w^n} - Z^*\beta_1 - \beta_2 E_{HT}] + U_2, \tag{21}$$

6 By (2) we have $\ln w_h^n = \ln w + \ln \text{RET}_h$, into which (9) and one of the expressions from (11) through (14) are substituted in turn to derive (16) through (19).

7 See Nakamura and Nakamura (1981) for details of deriving the expressions for (21), h^* and U_2 . Note that $\overline{\ln w^n}$ depends on h only through $\ln \text{RET}_h$, and $\ln w^n$ in the right-hand side of (21) is evaluated at $h = h^*$.

where $\overline{\ln w^n}$ is the deterministic part of $\ln w^n$ with

$$\overline{\ln w^n} = \overline{\ln w} + \ln \text{RET}_{h^*} \tag{22}$$

and

$$\overline{\ln w} = \alpha_0 + Z\alpha_1 + R\alpha_2. \tag{23}$$

The random term U_2 is some normal variable with $E(U_2) = 0$, and h^* is the solution to $h^* = (1/\beta_4)[- \beta_0 + (1 - \beta_3) \overline{\ln w^n} - Z^*\beta_1 - \beta_2 E_{HT}]$, which contains h^* on the right-hand side in a non-linear manner through $\ln \text{RET}_{h^*}$, which is one component of $\overline{\ln w^n}$. Assuming that the disturbance terms of (9) and (20) obey conditions (A2) in the Appendix and using the results presented there, we see that the equations to be estimated are

$$\ln w = \alpha_0 + Z\alpha_1 + R\alpha_2 + (\sigma_{12}/\sigma_2)\lambda + V \tag{24}$$

and

$$h = (1/\beta_4)[- \beta_0 + (1 - \beta_3) \overline{\ln w^n} - Z^*\beta_1 - \beta_2 E_{HT}] + \sigma_2\lambda + V^*, \tag{25}$$

where

$$\lambda = [f(\phi_{1400}) - f(\phi_0)]/[F(\phi_0) - F(\phi_{1400})] \tag{26}$$

for wives with $0 < h < 1,400$, and

$$\lambda = -f(\phi_{1400})/[F(\phi_{1400}) - 1] \tag{27}$$

for wives with $h \geq 1,400$, where f and F are, respectively, the standard normal and cumulative normal density functions. Our null hypothesis, therefore, is that the population values of the coefficients of (24) and (25) are the same for working wives who work less than 1,400 hours as they are for wives who work at least 1,400 hours. For both wives with $0 < h < 1,400$ and wives with $h \geq 1,400$ the means and the covariance structure of V and V^* are given by (A9) with V and V^* substituted for V_1 and V_2 in these conditions. The offered wage equation (24) can be estimated by OLS or GLS, and either OLS or GLS estimates can be obtained for the coefficients of the hours equation (25) by the following iterative algorithm, which preserves the non-linearity of $\ln \text{RET}$. (Note that the deterministic part of $\ln w$ is now given by $\overline{\ln w} = \alpha_0 + Z\alpha_1 + R\alpha_2 + (\sigma_{12}/\sigma_2)\lambda$.)

Given estimates of the log of the offered wage rate for each married woman, $\widehat{\overline{\ln w}}$, the hours equation (25) can be estimated iteratively using the following procedure described more fully in Nakamura and Nakamura (1981). We begin with $k = 0$ and $\ln \text{RET}^{(0)} = 0$ for Canada. For the United States we begin with $k = 0$ and $\ln \text{RET}_h^{(0)}$ calculated for $\widehat{h}^{(0)} = 1$. Using the resulting estimates for $\widehat{\overline{\ln w_{(0)}^n}}$, OLS or GLS estimates for $\widehat{h}^{(1)}$ are obtained from (25). These in turn are used to compute values for $\ln \text{RET}_h^{(1)}$ and $\widehat{\overline{\ln w_{(1)}^n}}$ and new OLS and GLS estimates for h , denoted by $\widehat{h}^{(2)}$, are

obtained from (25).⁸ This iterative process is continued until two successive sets of estimates for the coefficients of (25) are sufficiently close to each other in terms of percentage changes.

For the data samples used in this study, however, when the estimates for $\ln w$ are obtained by applying either OLS or GLS to (24) severe multicollinearity problems result in equation (25) among $\hat{\lambda}$, $\widehat{\ln w^n}$ and Z^* . This type of problem involving the selection bias term is also briefly discussed by Smith (1980, 22–3). Thus in this study we use a Durbin rank instrument to obtain the values of $\overline{\ln w}$ ⁹ used in estimating our hours equation (25).

OUR DATA AND THE VARIABLES INCLUDED IN OUR MODEL

The basic Canadian data consist of 27,401 records for married couples living in Canada, where the husband is 25–54 years old and no non-relatives are present, which are contained in the 1 per cent Family File of the Public Use Sample from the 1971 Canadian Census.¹⁰ The basic u.s. data consist of 27,414 records for married couples living in the United States, where the husband is 25–54 years old and no non-relatives are present, which are contained in the 1 per cent subsample from the 5 per cent primary State Public Use Sample of Basic Records from the 1970 u.s. Census.¹¹ The records for each country have been divided into five groups according to the child status of the wife. Separate results are presented for each country for (1) wives whose only children living at home are younger than 6, (2) wives with both children younger than 6 and children 6–14 at home, (3) wives whose only children living at home are 6–14, (4) wives with children ever born but no children younger than 15 at home, and (5) wives with no children ever born.

We define the vectors Z^* , Z , and R as follows:

Personal characteristics affecting a wife's asking wage (Z^)*

- Z^*1 . Number of children younger than 6 (included for wives with children younger than 6 only and for wives with children younger than 6 and 6–14) (+)
- Z^*2 . Number of children 6–14 years of age (included for wives with children 6–14 only and for wives with children younger than 6 and 6–14) (+)

8 The u.s. federal tax tables for 1969 (see Internal Revenue Service, 1971) and the state income tax tables (see Advisory Commission on Intergovernmental Relations, 1969) are closely followed to compute the values of $\ln RET^{(k)}$ for each u.s. wife. Canadian federal and provincial tax tables for 1970 (see Department of National Revenue, 1972) and the Quebec income tax table for 1970 (see Gilmour, 1968) are used to compute the values of $\ln RET^{(k)}$ for each Canadian wife. See Nakamura and Nakamura (1981, 473–5) for details.

9 The properties of Durbin's rank instrument method are discussed in Johnston (1972, 283–6) and Kendall and Stuart (1973, 424, 529). For our purpose of predicting $\ln w$ the assumptions we require are that the ranking of our observations on $\ln w$ is determined by the ranking of the (unobservable) values of $\overline{\ln w}$, and that the predicted values of $\overline{\ln w}$, resulting from the regression of $\ln w$ on the rank of w , converge in probability to $\ln w$.

10 See Statistics Canada (1975).

11 See u.s. Bureau of Census (1972).

- Z*3. Number of children ever born (included for wives with children ever born but no children younger than 15) (+)
- Z*4. Employment income of the husband plus asset and other non-employment income of the family (excluding the employment income of the wife), net of federal and provincial or state income taxes to be paid at zero hours of work for the wife; measured in thousands of dollars, and denoted as E_{HT} above. For convenience this variable will be referred to as 'other income' (+)
- Z*5. Religion dummy (= 1 if religion of wife is Roman Catholic, = 0 otherwise; for Canada only) (+)
- Z*6. Language dummy (= 1 if language of home is French, = 0, otherwise; for Canada only) (+)
- Z*7. Race dummy (= 1 if wife is black,¹² = 0 otherwise; for United States only) (?)
- Z*8. Age of wife; measured in tens of years (+)

Personal characteristics affecting a wife's offered wage (Z)

- Z1. Age of wife; measured in tens of years (?)
- Z2. Age of wife at first marriage (+)
- Z3. Wife's years of education (+)
- Z4. Race dummy (= 1 if wife is black, = 0 otherwise; for United States only) (-)

Regional economic variables affecting a wife's offered wage (R)

- R1. Regional index of potential hours of work for wives in different age and educational categories; denoted in the text by HI. (+)

The plus and minus signs in parentheses following the variable names indicate the expected impact of each variable on the wife's asking or offered wage. It should be noted that the coefficients of the variables appearing in the offered wage function (Z and R) are expected to have the same signs in our probit equations as in the offered wage function, while the coefficients of the variables appearing in the asking wage function (Z^*) are expected to have the opposite signs in our probit and hours of work equations as in our asking wage function. In assuming that the child status variables should have a positive impact on the asking wage, we are implicitly assuming that the child-related costs of a wife's working rise more rapidly as the number of children is increased than the perceived need for increased income as family size is increased. As in Nakamura and Nakamura (1981), it is also assumed that the asking wage is a positive function of both the log of the marginal after-tax offered wage and hours of work (that is, $\beta_3 > 0$ and $\beta_4 > 0$ in (8) and hence in (20)), and it is assumed that the asking wage does not depend on the wife's education.

Of the variables included in this study, we shall discuss only the hours index, HI, since the other variables were all used in Nakamura and Nakamura (1981). An index

¹² Actually, information is available only in the U.S. census data for whether the husband is black, in which case we assume for lack of further information that the wife is also black.

designed to capture the impact on the offered wage of the expected job opportunities for women relative to the size of the potential female labour force was also used in this earlier study. This index performed well empirically. Nevertheless an index based on numbers of jobs cannot reflect changes in the tightness of a labour market resulting from changes in the annual number of hours of work available per job. Our new hours index is designed to overcome this problem.

The values of our new hours index were calculated as follows:

$$HI_{j,l} = \frac{\sum_{k=1}^K p_{jk} q_{k,l} H_l}{\sum_{j' \in j} P_{j'} N_{j',l}},$$

where H_l is the total hours of work in locality l , $q_{k,l}$ is the expected proportion of the total hours of work in locality l in occupation k , p_{jk} is the expected proportion of the total hours of work in occupation k going to workers of type j , $N_{j',l}$ is the total number of women of type j' in locality l , and $P_{j'}$ is the expected employment rate for women of type j' . The localities are defined by province and place of residence (urban $\geq 30,000$, urban $< 30,000$, rural) for Canada, and by state and place of residence (urban, rural, entire state where urban/rural distinction cannot be made) for the United States. The values of $q_{k,l}$ were calculated as the actual local proportions of the total hours of work in each occupation, using the same occupational groupings as are used in Nakamura and Nakamura (1981). Hence in this study $q_{k,l} H_l$ is the actual hours of work in locality l in occupation k . The values of p_{jk} were calculated as the national proportions of the total hours of work in each place of residence in occupation k going to workers of type j . Thus the numerator of $HI_{j,l}$ is the expected total number of hours of work in locality l going to workers of type j . The index j is used to denote nine types of women defined by the combinations of three age groups (15–24, 25–54, 55+) and three educational groups (≤ 8 years, 9–13 years, some university). The index j' denotes the relevant subdivisions *within* each of our nine basic groups, where these subdivisions are with respect to age (15–19, 20–24, 25–29, ... , 55–59, 60+), marital status (single; married; widowed, divorced, or separated), and child status (none ever born, 1 ever born, 2 ever born, 3+ ever born). After the indicated aggregation, therefore, the denominator of $HI_{j,l}$ represents the expected number of women of each of our nine basic types in each locality who will work based only on the stated demographic characteristics of these women. In other words, for women of a given type the denominator in question is a refined measure of the potential labour force. The index HI is thus a measure of the expected annual hours of work per potential working woman of a given type in a given locality. It is expected that higher values of HI will be associated with higher values of the offered wage rates.

EMPIRICAL RESULTS

In tables 1 and 2 we show OLS estimates for the coefficients of our offered wage and annual hours of work equations for both Canada and the United States for each of our

five child status groups.¹³ To obtain the results labelled CAN for Canada for (24) and (25) we included data for all wives with positive employment income for 1970 and for whom $0 < h_i < 1,400$ for the results shown in the left-hand column for each child status group, and for whom $h_i \geq 1,400$ for the results shown in the right-hand column for each child status group. To obtain the results labelled US for the United States for (24) and (25) we included data for all wives with positive employment income in 1969 who also worked in the census reference week and for whom $0 < h_i < 1,400$ for the results shown in the left-hand column for each child status group, and for whom $h_i \geq 1,400$ for the results shown in the right-hand column for each child status group. In order to obtain the values for the selection bias term, λ , included in (24) and (25) the coefficients of (16) through (19) were estimated using probit analysis. Next values of ϕ_0 and ϕ_{1400} were calculated for each wife using the estimated versions of (16) and (18) for Canadian wives who file separate tax returns if they work, and using the estimated versions of (17) and (19) for U.S. wives who file joint tax returns if they work. Finally the appropriate value of λ was calculated for each working wife using formula (26) for wives with $0 < h < 1,400$, and formula (27) for wives with $h \geq 1,400$. In estimating the coefficients of (16) and (18), the dummy dependent variable for the probit analysis was set equal to 1 if $h > 0$, and the wife earned positive employment income for 1969 for the United States and for 1970 for Canada. In estimating the coefficients of (17) and (19) the dummy dependent variable for the probit analysis was set equal to 1 if $h \geq 1,400$ and the wife's earned positive employment income for the relevant calendar year.¹⁴ The full data sample for Canada including both working and non-working wives was used in estimating the coefficients of (16) and (18), and the full data sample for the United States was used in estimating the coefficients of (17) and (19).

The signs of the coefficients of (24) and (25) shown in tables 1 and 2 are generally in agreement with our expectations, and with the results shown in Nakamura and Nakamura (1981).

Fuchs has suggested that the real, rather than the nominal, asking and offered wage rates should be viewed as functions of the variables included in Z^* , Z , and R , with the after-tax other income of the family expressed in real terms as well. His joint working paper with Michael and Scott (1979) contains a state price index that can be used to implement his suggestion for the U.S. portion of our analysis. We began our

13 For the particular data samples used in this study we find that the regressions that must be performed in order to obtain GLS estimates – of the squares of the OLS residuals for the offered wage equation (24) on a constant term and M – to be very insignificant. (See the appendix for a description of the procedures for obtaining GLS estimates and the definition of M .) Thus in this paper OLS estimates are presented for the parameters of (24) and (25).

14 For Canada the values of h_i were computed by multiplying the number of weeks the wife worked in 1970 times her usual number of hours worked per week for the job held in the reference week for the 1971 Canadian census, or otherwise for the job of longest duration since 1 January of the previous year. For the United States the values of h_i were computed by multiplying the number of weeks worked in 1969 times the actual number of hours worked at all jobs in the census reference week. Owing to the missing data problem noted in the text, we have no way of directly computing annual hours of work for U.S. wives who were not 'at work' during the census reference week. Thus the λ values for U.S. wives account not only for selectivity based on actual annual hours of work, but also for the potential selectivity bias resulting from this missing data problem for U.S. wives.

TABLE 1 (concluded)

Lamda (λ)	CAN	Children < 6		Children < 6 and 6-14		Children 6-14		None < 15		None ever born		Significant difference for coefficients for $0 < h < 1,400$ compared with $h \geq 1,400^c$
		$0 < h < 1,400$	$h \geq 1,400$	$0 < h < 1,400$	$h \geq 1,400$	$0 < h < 1,400$	$h \geq 1,400$	$0 < h < 1,400$	$h \geq 1,400$	$0 < h < 1,400$	$h \geq 1,400$	
Constant		0.213** (0.078)	-0.066 (0.204)	-0.244** (0.129)	0.473 (0.601)	-0.186** (0.086)	-0.149 (0.140)	0.010 (0.165)	-0.881** (0.183)	-0.806** (0.355)	0.039 (0.188)	
	US	0.028 (0.129)	-0.157 (0.167)	-0.188** (0.100)	0.066 (0.126)	-0.103* (0.078)	-0.126 (0.082)	-0.471** (0.146)	-0.069 (0.098)	0.074 (0.218)	-0.475** (0.176)	
	CAN	-0.522** (0.194)	-0.468** (0.137)	-0.321 (0.300)	-0.696** (0.350)	-0.126 (0.191)	0.082 (0.147)	-0.029 (0.317)	0.891** (0.199)	-0.324* (0.247)	-0.232** (0.169)	Yes
	US	-0.701** (0.343)	-0.243* (0.179)	-0.351 (0.278)	-0.425** (0.200)	-0.215 (0.193)	-0.004 (0.109)	-0.418* (0.280)	-0.262* (0.137)	-0.787** (0.263)	-0.211 (0.179)	Yes
R^2	CAN	0.122	0.263	0.049	0.088	0.033	0.109	0.041	0.144	0.131	0.184	
	US	0.084	0.103	0.060	0.105	0.038	0.079	0.079	0.106	0.156	0.170	
Number of observations	CAN	1364	639	1150	479	2053	1692	788	1025	690	1734	
	US	486	467	742	579	1627	2046	810	1813	432	1199	

SOURCE: Calculated from the 1 per cent Family File of the Public Use Sample from the 1971 Canadian Census; and from the 1 per cent subsample from the 5 per cent primary State Public Use Sample of Basic Records from the 1970 U.S. Census.

^aThe data sets for Canada (CAN) consist of wives in the indicated child-status and annual hours of work categories who earned employment income in 1970. The data sets for the United States (US) consist of wives in the indicated child-status and annual hours of work categories who earned employment income in 1969 and who also worked in Census reference week.

^bNumbers in parentheses are the appropriate (asymptotic) standard deviations under the null hypothesis of no selection bias. Significance levels indicated are: at least 95 per cent for coefficients with two asterisks and at least 80 per cent for coefficients with one asterisk.

^cUsing critical region of 20 per cent. See text for details.

TABLE 2 (concluded)
OLS estimates for annual hours of work equation for married women in Canada and the United States, classified by child status and annual hours of work (h)^{a, b}

	Children < 6 $0 < h < 1,400$	Children < 6 and 6-14 $0 < h < 1,400$	Children 6-14 $0 < h < 1,400$	None < 15 $0 < h < 1,400$	None ever born $0 < h < 1,400$	Significant difference for coefficients for $0 < h < 1,400$ compared with $h \geq 1,400$ ^c
Language dummy (= 1 if language of home is French; = 0 otherwise)						
CAN	28.93 (36.41)	-26.53 (27.75)	-36.39 (38.74)	105.35* (65.06)	40.00 (46.92)	Yes
US						
Race dummy (= 1 if wife is black; = 0 otherwise)						
US	0.91 (106.44)	-38.39 (43.58)	14.76 (42.55)	88.37* (55.03)	53.59 (60.86)	Yes
CAN	69.30 (60.11)	-282.59** (80.61)	92.00* (48.66)	107.97* (74.17)	116.21** (43.80)	Yes
US	174.77 (149.16)	-5.97 (104.82)	-40.61 (75.53)	290.73 (400.86)	265.85** (66.58)	Yes
CAN	798.35** (43.44)	1948.63** (95.25)	1796.72** (144.58)	801.31** (51.78)	717.32** (52.50)	Yes
US	932.55** (75.92)	2033.56** (102.28)	2128.07** (114.33)	815.09** (50.13)	785.18** (70.60)	Yes
R ²	0.054 0.090	0.079 0.081	0.069 0.069	0.085 0.071	0.046 0.055	0.087 0.019
Mean annual hours of work	593 671	1921 1952	610 734	652 822	782 886	1950 1984

SOURCE: Calculated from the 1 per cent Family File of the Public Use Sample from the 1971 Canadian Census; and from the 1 per cent subsample from the 5 per cent primary State Public Use Sample of Basic Records from the 1970 U.S. Census.

^aThe data sets for Canada (CAN) consist of wives in the indicated child-status and annual hours of work categories who earned employment income in 1970. The data sets for the United States (US) consist of wives in the indicated child-status and annual hours of work categories who earned employment income in 1969 and who also worked in Census reference week.

^bNumbers in parentheses are the appropriate (asymptotic) standard deviations under the null hypothesis of no selection bias. Significance levels indicated are: at least 95 per cent for coefficients with two asterisks and at least 80 per cent for coefficients with one asterisk.

^cUsing critical region of 20 per cent. See text for details.

investigation of this question by first including the log of this price variable as an explanatory variable in nominal asking and offered wage equations. We re-estimated the resulting probit, offered wage and hours equations for the United States, and tested the hypothesis that the coefficient of the log of the price variable in the offered wage equation is 1. This hypothesis can be accepted at a 95 per cent confidence level for all our child status and hours of work groups for the United States except wives with children 6–14 working more than 1,400 hours. The *t*-statistic for this group is 2.13. Based on these results, we then re-estimated our model for the United States using a real specification for our offered and asking wage equations. In particular, the other income of the family was expressed in real terms, and the dependent variable of the offered wage equation was expressed as the log of the offered wage minus the log of the price variable; but the log of the price variable was no longer included as an explanatory variable in our probit and hours equations. Comparing these results with those shown in tables 1 and 2, the coefficient estimates were found to be virtually identical except for the expected changes in magnitude in the estimates of the constant term of the offered wage equation and the coefficient of the other income variable in the hours equations. The reason for this similarity is not that the price differentials between states are unimportant, but rather that the price index is virtually uncorrelated with our other explanatory variables. Hence it appears that our results presented in tables 1 and 2 of this paper are not seriously biased because regional price differentials were ignored in obtaining these results.

We turn now to the central question of whether there is evidence of systematic differences in the values of the coefficients of (24) and (25) for wives working less than 1,400 hours compared with the values of these coefficients for wives working at least 1,400 hours. For each country and each hours of work category, estimates of the coefficients of (24) and (25) have been obtained for wives in five child-status categories. Under our null hypothesis that the population values of the coefficients of (24) and (25) are the same for wives working less than 1,400 hours as for wives who work at least 1,400 hours, the probability of the estimated value of a coefficient being consistently higher (or consistently lower) for five out of five child-status groups for part-time versus full-time wives is 3.13 per cent, while the probability of the estimated value of a coefficient being consistently higher (or consistently lower) for four out of five child-status groups for part-time versus full-time wives is 15.6 per cent.¹⁵ Looking at tables 1 and 2, using a 20 per cent critical region we find evidence of systematic differences in the values of the coefficients of our wage equation for part-time versus full-time wives for two out of six coefficients for Canadian wives and for three out of seven coefficients for U.S. wives; and evidence of systematic differences in the value of the coefficients of our hours equation for part-time versus full-time wives for five out of six coefficients for Canadian wives and for four out of five coefficients for U.S. wives.¹⁶ Thus we do find evidence of parameter instability in

15 These probabilities are calculated using the binomial distribution with a success defined as the coefficient estimate for wives with $0 < h < 1,400$ exceeding the coefficient estimate for wives with $h \geq 1,400$, $n = 5$ and $p = 0.5$.

16 The child-status variables that are not included in all five of the child-status groupings for wives in each country are not included in this count.

the empirical model given by (24) and (25) over the range of variation in annual hours of work.

SOME IMPLICATIONS OF OUR RESULTS

Whether or not parameter instability of the sort we have identified is of substantive importance will depend on the intended use of the empirical results. If we compare the part-time and full-time coefficient estimates for those cases where both are significant with even an 80 per cent level of confidence, we find only one instance where the signs of the two coefficients estimates differ. Thus we do not find that our inferences about coefficient signs would be altered by accounting for this parameter instability. Of particular interest, we also find no evidence that the signs of the uncompensated wage elasticities of hours of work are different for part-time versus full-time working wives. Rather, we find these elasticities to be negative for both classifications of working wives.

Angus Deaton and John Muellbauer (1980, 276) write that 'For men, whose shadow wage is low and who work relatively long hours, the income effect is dominant so that the labor supply curve is backward sloping, at least in the observed range. For women, however, the high value of time spent in the home sets a relatively high shadow wage and both participation and hours at work are lower than for men. With shorter working hours, the income effect is necessarily relatively unimportant, so that rising real wages account for greater labor supply by married women through increased participation and longer hours.' Using a similar line of reasoning, it has also been suggested that the labour supply curve may be backward sloping for married women who work full time, but that wives who work part time will increase their labour supply when the real wage rises. If this hypothesis is correct, we should find in table 2 that the estimated coefficients of the log of the hourly wage variable are systematically more positive for wives who work less than 1,400 hours than for those who work at least 1,400 hours. In table 2, however, we find no evidence of a systematic difference in the value of the coefficient of the wage variable depending on the hours of work category for Canadian wives, and for U.S. wives we find that the estimated coefficients of the wage variable are systematically more negative for part-time compared with full-time working wives. The corresponding uncompensated wage elasticities of annual hours of work are shown in table 3. They are seen to lie in the range of values reported in Nakamura and Nakamura (1981, 483) for married women, and in the range of values reported by other researchers for men. These results lend no support then to the hypothesis that the labour supply curve (conditional on working) is backward sloping for wives working long hours, but not for wives working shorter hours.

The wage rate variable used in this study and in Nakamura and Nakamura (1981) is created by dividing reported annual income for the relevant calendar year by a measure of annual hours of work. The measure of annual hours of work is in turn created by multiplying weeks of work in the relevant calendar year by hours of work in the census reference week for U.S. wives, and by 'usual' hours of work per week for Canadian wives. If our measure of annual hours of work is erroneously high (or low)

TABLE 3
Uncompensated wage elasticities by child status and annual hours of work for Canada and the United States^a

	Child status and hours of work of wife									
	Children < 6 0 < h < 1,400	h ≥ 1,400	Children < 6 and 6-14 0 < h < 1,400	h ≥ 1,400	Children 6-14 0 < h < 1,400	h ≥ 1,400	None < 15 0 < h < 1,400	h ≥ 1,400	None ever born 0 < h < 1,400	h ≥ 1,400
Canada	-0.108	-0.048	-0.128	-0.040	-0.197	-0.036	-0.179	-0.054	-0.083	-0.087
United States	-0.176	-0.031	-0.178	-0.049	-0.163	-0.030	-0.145	-0.033	-0.120	-0.037

SOURCE: Calculated from the 1 per cent Family File of the Public Use Sample from the 1971 Canadian Census; from the 1 per cent subsample from the 5 per cent primary State Public Use Sample of Basic Records from the 1970 U.S. Census; and from the coefficient estimates of the Log of predicted hourly wage variable shown in table 2 for CAN and US
^aEvaluated at mean hourly wage rate and annual hours of work for each group

for a given wife then our computed wage rate will tend to be erroneously low (or high). If the computed wage rate variable were entered directly (in log form) into our equation for the log of the wife's annual hours of work, this errors-in-the-variables problem would lead to a negative bias on the coefficient of the wage rate variable. It is plausible also that the errors-in-the-variables problem for our measure of annual hours of work is more serious for those who work irregularly or who work only a few hours per week than for full-time workers. Thus if the computed wage variable were entered directly into our equation for the log of the wife's annual hours of work, there might well be a larger negative bias on the coefficient of the wage variable when this coefficient was estimated using data from a sample of wives with $0 < h < 1,400$ than when this coefficient was estimated using data from a sample of wives with $h \geq 1,400$.

This is not what we have done, however; in both Nakamura and Nakamura (1981) and in the present study an instrumental variable has been used for the wage rate variable appearing in the equation for the log of the wife's annual hours of work. Thus the errors-in-the-variables problem discussed above can 'explain' our finding of a negative relationship between the wage rate and annual hours of work and our finding of a more negative relationship between the wage rate and annual hours of work for part-time than for full-time working wives, only if some portion of the measurement error in question is systematically picked up by the instrument for the wage rate. In this respect, it should be noted that the Durbin rank instrument used in this study is quite different from the reduced form predictions of the log of the offered wage rate used as an instrument in Nakamura and Nakamura (1981). Also the errors of measurement problem should be different for the United States than for Canada, since 'usual' hours of work per week are reported in the Canadian census while actual hours of work in the census reference week are reported in the U.S. census. Yet comparing the computed (uncompensated) elasticities of hours of work with respect to the after-tax offered wage rate, we find great similarity between the values reported in this study (table 3) and the values reported in Nakamura and Nakamura (1981, 483, table xi), and between the respective sets of estimates for the two countries. Thus, although we are continuing to investigate this question, we do not believe that our results in this paper point to an errors-in-the-variables problem.

We feel it is more likely these results mean that the underlying theoretical model should be revised, or at least reinterpreted. Perhaps many families determine the long-term level of family consumption based solely on the husband's earning capacities, and many of the wives found working part-time are simply working to cover what the family regards as short-term divergences between actual levels of consumption and the husband's earnings. The income effect on hours of work of a wage change will be very strong, of course, for a wife who is working solely to pay for some specific item such as a vacation, an addition to the family home, or the education of a child. We hope that the responses obtained to some of the questions that have been asked in the Michigan Panel Study of Income Dynamics will shed some light on these questions. On the basis of the present study, the hypothesis that the coefficient of the wage variable in the equation for the wife's annual hours of work is the same for wives with $0 < h < 1,400$ as for wives with $h \geq 1,400$ is rejected for

the United States and the hypothesis that this coefficient is more positive for wives with $0 < h < 1,400$ than for wives with $h \geq 1,400$ is also rejected for both the United States and Canada. Further investigation is now required to gain a substantive understanding of these results.

Obtaining separate sets of estimates of the coefficients of our wage rate and hours of work equations for part-time and full-time working wives also provides us with some additional insight into the nature of the labour market disadvantage of black wives in the United States. Nakamura and Nakamura (1981) find the estimated coefficients of the race dummy in the offered wage equation to be consistently negative and generally highly significant, with the values of the coefficient estimates ranging from -0.044 to -0.193 . In the present study the coefficient estimates of the race variable in the offered wage equation are always *insignificant* for wives with $0 < h_i < 1,400$, except for wives with children younger than 6. For wives working more than 1,400 hours who have had children, the estimated coefficients for this variable are always negative and generally significant, and range from -0.046 to -0.138 . For wives working full time with no children ever born we estimate this coefficient to be -0.302 , and it is highly significant. These full-time black wives, therefore, are seen to be bearing the brunt of the wage rate differential between black and non-black wives, which cannot be attributed to racial differences in the distributions of measurable personal characteristics such as years of education. Based on these results, we cannot agree completely with Freeman's (1973, 280) conclusion concerning race discrimination that, 'for women ... discriminatory differences appear to have virtually disappeared.' Moreover part of the dramatic 'collapse' in the economic differences separating black and white women documented by Freeman (1973, 280 and 281, table 1) may well be due to major shifts over this same time period in the proportions of black and non-black women working part versus full time.¹⁷ Even quite small differences in coefficient values over the range of variation in annual hours of work might substantively affect the results in other studies of discrimination where discrimination is measured as the residual difference after accounting for differences between the group of interest and the comparison group in the values of the explanatory variables.

A final finding of this study is that it is extremely important to account for parameter differences, particularly in the values of the constant term, between wives found to work part time and those found to work full time in equations to be used for predicting the hours of work of individual wives. In order to explore this point more fully we re-estimated our annual hours equation (25) using data for all the wives found to work in each child status category for each country. Thus we now have

$$\lambda = f(\phi_0)/F(\phi_0), \quad (28)$$

as in Nakamura and Nakamura (1981). In table 4 we show the distributions of actual

¹⁷ Freeman (1973, 280) finds a 'collapse in the economic differences separating black and white women, with the ratio of incomes rising from 0.50 (1950) to 0.86 (1970).' For a very interesting discussion of how shifts in the proportions of black and white women working full- versus part-time may have affected the black / white female wage differential, as well as discussions and evidence concerning other related issues, see James P. Smith (forthcoming).

TABLE 4
Actual and predicted distributions for annual hours of work and R^2 s

Annual hours of work	Canada			United States		
	Actual	Hours equations allowing for selection into category of $h > 0$	Hours equations allowing for selection into categories of $0 < h < 1,400$ and $h \geq 1,400$	Actual	Hours equations allowing for selection into category of $h > 0$	Hours equation allowing for selection into categories of $0 < h < 1,400$ and $h \geq 1,400$
$0 < h \leq 600$	28	0	25	14	0	8
$600 < h \leq 1,200$	13	18	25	18	4	29
$1,200 < h \leq 1,800$	17	51	0	20	53	0
$1,800 < h \leq 2,200$	35	20	48	41	29	60
$h > 2,200$	4	13	1	7	15	0
R^2 for regression of predicted on actual individual values		0.064	0.810		0.046	0.779
Number of observations		11,614			10,201	

hours of work; of predicted hours of work when we use the hours equations for which coefficient estimates are shown in table 2, allowing for selection into the part-time and full-time categories of $0 < h < 1,400$ and $h \geq 1,400$; and of predicted hours when we use our hours equations allowing only for selection into the category of workers with $h > 0$. In obtaining the predicted values for annual hours we used the observed wage rates net of taxes evaluated at the wife's actual hours of work.

From table 4 we see that the mode of the predicted values for annual hours of work obtained allowing only for selection into the category of workers ($h > 0$) lies in the range of $1,200 < h \leq 1,800$, while the modes of the actual values and of the predicted values allowing for selection into the categories of $0 < h < 1,400$ and $h \geq 1,400$ are in the range of $1,800 < h \leq 2,200$. The distributions of predicted hours allowing for selection into part-time and full-time work categories are not entirely satisfactory either. The distributions are too concentrated in the modal range of $1,800 < h \leq 2,200$, and the shape of the distribution of actual hours for those working less than 1,800 hours is not properly captured for either country. The R^2 s at the bottom of table 4 for the regressions of the predicted on the actual individual annual hours of work make it clear, however, that there is a dramatically better correspondence between the actual values and the predicted values obtained by first sampling into our part-time and full-time hours of work categories than between the actual values and the predicted values obtained allowing only for sampling into the category of workers.

The distributions of prediction errors are also quite different for our two methods of predicting annual hours of work. All our hours of work equations were estimated using a Durbin rank instrument for the included wage rate variable and iterated using hours of work from the previous iteration to determine the tax rate. In obtaining

TABLE 5
Summary statistics for the distributions of prediction errors

	Canada		United States	
	Hours equations allowing for selection into category of $h > 0$	Hours equations allowing for selection into categories of $0 < h < 1,400$ and $h \geq 1,400$	Hours equations allowing for selection into category of $h > 0$	Hours equations allowing for selection into categories of $0 < h < 1,400$ and $h \geq 1,400$
Mean prediction error	-374.50	7.36	-289.12	-2.43
Standard deviation for prediction errors	778.77	330.20	709.17	325.68
Test statistic for test of null hypothesis that the distribution of prediction errors has a zero mean ^a	-51.82	2.40 ^b	-41.18	-0.75 ^b

^aCritical values are 1.960 for a 95 per cent level of confidence, 2.326 for a 98 per cent level of confidence, and 2.576 for a 99 per cent level of confidence using a two-tailed test.

^bNull hypothesis accepted with a critical region of 0.01.

values for predicted hours of work, however, we have used the observed wage rates net of taxes evaluated at the wife's observed hours of work. Thus there is no definitional reason why the distributions of prediction errors must have zero means. We see from table 5 that the null hypothesis that the distribution of prediction errors has a zero mean is rejected for both countries when the predictions for annual hours are obtained by the conventional method allowing only for sampling into the category of workers. However, this null hypothesis is accepted for both countries using a critical region of 0.01 when the predictions for hours of work are obtained allowing for sampling into our part-time and full-time categories for hours of work. From table 6 we see also that the distributions of prediction errors for the latter method are much more symmetrical and closer in shape to a normal distribution with zero mean and the observed sample variance than is the case for the former method. The null hypothesis that the prediction errors obey a normal distribution is, however, rejected for both methods for both countries using a conventional χ^2 test.

CONCLUSIONS

In summary, we find statistical evidence of parameter instability in a conventional empirical model of the labour force behaviour of married women over the range of variation in annual hours of work. This instability is not severe enough to result in sign changes. Nor do we find any evidence that the uncompensated wage elasticities of hours of work are positive for wives working part time as has been hypothesized. In fact, we find these elasticities to be consistently negative for working wives in both

TABLE 6

Normal distribution and distributions of prediction errors

	Canada			United States		
	Normal distribution with zero mean and standard deviation of 330.20	Hours equations allowing for selection into category of $h > 0$	Hours equations allowing for selection into categories of $0 < h < 1,400^a$ and $h \geq 1,400^a$	Normal distribution with zero mean and standard deviation of 325.68	Hours equations allowing for selection into category of $h > 0$	Hours equations allowing for selection into categories of $0 < h < 1,400$ and $h \geq 1,400^b$
< -400	0.11	0.48	0.10	0.11	0.40	0.10
-400 to -200	0.16	0.06	0.18	0.16	0.08	0.16
-200 to 200	0.46	0.19	0.50	0.46	0.23	0.51
200 to 400	0.16	0.10	0.08	0.16	0.12	0.12
>400	0.11	0.16	0.14	0.11	0.17	0.10

^aStandard deviation of prediction errors equals 330.20.

^bStandard deviation of prediction errors equals 325.68.

hours of work categories. We do find, however, that accounting for parameter instability of the sort identified may be important in making labour force comparisons between different groups such as blacks and non-blacks or women and men. And accounting for this sort of instability, particularly in the constant term, is found to be particularly important when equations estimated from cross-sectional data, in which it is not possible to take account of fixed or persistent individual effects, are to be used for predicting hours of work for large numbers of individual wives.

APPENDIX: A TWO-STAGE ESTIMATION METHOD
FOR DATA SAMPLES CENSORED AT TWO LIMITS

We are interested in estimating β_1 and β_2 where the reduced form equations for individual i ($i = 1, \dots, I$) are

$$Y_{1i} = X_{1i}\beta_1 + U_{1i}, \quad (\text{A1a})$$

and

$$Y_{2i} = X_{2i}\beta_2 + U_{2i}, \quad (\text{A1b})$$

and where X_{ji} ($j = 1, 2$) are $1 \times K_j$ vectors of exogenous regressors, β_j are $K_j \times 1$ vectors of parameters, and U_{1i} and U_{2i} are bivariate normal random variables such that

$$E(U_{1i}) = E(U_{2i}) = 0, \quad (\text{A2a})$$

$$E(U_{ji}^2) = \sigma_j^2 \quad \text{for } j = 1, 2, \quad (\text{A2b})$$

$$E(U_{1i}U_{2i}) = \sigma_{12}, \quad (\text{A2c})$$

and

$$E(U_{ji}U_{j'i'}) = 0 \quad \text{for } i \neq i' \quad \text{and } j = 1, 2. \quad (\text{A2d})$$

The first element of X_{1i} and of X_{2i} is assumed to be one. We assume that for individual i the value of Y_{1i} is observed if

$$a < Y_{2i} < b, \quad (\text{A3})$$

where a and b are distinct real numbers ($a < b$). Without loss of generality we assume that for the first I_1 ($\leq I$) individuals data is available on Y_{1i} . (Heckman's model (1976, 1977, 1979) is a special case of (A3) for which $a = 0$ and $b = \infty$. Also Poirier (1978) and Poirier and Melino (1978) consider a degenerate case in which Y_2 does not exist and hence Y_1 is observed if and only if $a < Y_1 < b$, which is again a special case of our model presented here.)

In the empirical sections of this paper, the i th individual is the i th wife,¹⁸ Y_{1i} corresponds to her offered (market) hourly wage, and Y_{2i} corresponds to her annual hours of work. The i th wife is considered to work part time if her annual hours of

18 The subscript i has been dropped in the body of this paper for notational convenience.

work are greater than zero and less than 1,400, and full time if her annual hours of work are at least 1,400. The offered wage is observed for both part-time and full-time working wives.

In general, the subsample regression functions are

$$E(Y_{1i}|X_{1i}, a < Y_{2i} < b) = X_{1i}\beta_1 + E(U_{1i}|a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2) \quad (\text{A4a})$$

and

$$E(Y_{2i}|X_{2i}, a < Y_{2i} < b) = X_{2i}\beta_2 + E(U_{2i}|a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2), \quad (\text{A4b})$$

where

$$E(U_{1i}|a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2) = (\sigma_{12}/\sigma_2)\lambda_i, \quad (\text{A5a})$$

$$E(U_{2i}|a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2) = \sigma_2\lambda_i, \quad (\text{A5b})$$

$$\lambda_i = (f(\phi_{bi}) - f(\phi_{ai})) / (F(\phi_{ai}) - F(\phi_{bi})), \quad (\text{A6})$$

$$\phi_{ai} = (1/\sigma_2)(a - X_{2i}\beta_2), \quad \phi_{bi} = (1/\sigma_2)(b - X_{2i}\beta_2), \quad (\text{A7})$$

and f and F are, respectively, the standard normal and cumulative density functions. Thus the conditional regression functions to be estimated for individuals satisfying (A3) may be written as

$$Y_{1i} = X_{1i}\beta_1 + (\sigma_{12}/\sigma_2)\lambda_i + V_{1i} \quad (\text{A8a})$$

and

$$Y_{2i} = X_{2i}\beta_2 + \sigma_2\lambda_i + V_{2i}, \quad (\text{A8b})$$

where

$$E(V_{1i}|X_{1i}, \lambda_i, a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2) = 0, \quad (\text{A9a})$$

$$E(V_{2i}|X_{2i}, \lambda_i, a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2) = 0, \quad (\text{A9b})$$

and

$$E(V_{ji}V_{j'i'}|X_{1i}, X_{2i}, \lambda_i, a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2) = 0 \quad \text{for } i \neq i' \text{ and } j = 1, 2, \quad (\text{A9c})$$

and where¹⁹

$$E(V_{1i}^2|X_{1i}, \lambda_i, a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2) = \sigma_1^2(1 - \rho^2) + \sigma_1^2\rho^2M_i, \quad (\text{A9d})$$

$$E(V_{2i}^2|X_{2i}, \lambda_i, a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2) = \sigma_2^2M_i, \quad (\text{A9e})$$

$$E(V_{1i}V_{2i}|X_{1i}, X_{2i}, \lambda_i, a - X_{2i}\beta_2 < U_{2i} < b - X_{2i}\beta_2) = \sigma_{12}M_i, \quad (\text{A9f})$$

19 We obtain these results using results found in Johnson and Kotz (1970, chap. 13, 77, 81, 83; chap. 36, 112).

for $\rho^2 = \sigma_{12}^2/(\sigma_1^2\sigma_2^2)$, $M_i = 1 + \xi_i - \lambda_i^2$, and

$$\xi_i = (\phi_{ai}f(\phi_{ai}) - \phi_{bi}f(\phi_{bi})) / (F(\phi_{bi}) - F(\phi_{ai})). \tag{A9g}$$

We also have

$$0 \leq 1 + \xi_i - \lambda_i^2 \leq 1 \tag{A10}$$

from (A9e) and from the fact that the maximum of $(1 + \xi_i - \lambda_i^2)$ with respect to ϕ_{ai} and ϕ_{bi} is 1. (Our expressions (A9d)–(A9f) correspond to Heckman’s expressions (4f)–(4h), and our (A10) corresponds to his (5).²⁰ It is easily seen that our expressions reduce to his if we set $a = 0$ and $b = \infty$.)

Probit analysis for the probability that $Y_{2i} > a$ gives estimates for the coefficients $(\beta_{21} - a)/\sigma_2, \beta_{22}/\sigma_2, \dots, b_{2K_2}/\sigma_2$; and probit analysis for the probability that $Y_{2i} > b$ gives estimates for the coefficients $(\beta_{21} - b)/\sigma_2, \beta_{22}/\sigma_2, \dots, \beta_{2K_2}/\sigma_2$. The entire sample is used in both cases with the dummy dependent variable set equal to 1 if the appropriate condition on Y_{2i} is satisfied for the i th individual and set equal to 0 otherwise. These estimated coefficients can be used to calculate $\hat{\phi}_{ai}, \hat{\phi}_{bi}, \hat{\lambda}_i$, and \hat{M}_i , the estimated values for $\phi_{ai}, \phi_{bi}, \lambda_i$, and M_i for each individual. The conditional regression functions (A8a) and (A8b) may then be estimated directly for the appropriate subsamples of individuals using OLS and ignoring (A9d)–(A9f). Or we may recover the residuals from these OLS regressions, and regress the squares of the residuals from (A8a) on a constant term and \hat{M}_i to obtain estimates of the variance of V_{1i} for each individual in a particular subsample. Estimates of the variance of V_{2i} for each individual in the subsample are given by $S_2^2\hat{M}_i$, where S_2 is the OLS coefficient of λ_i in (A8b). These variance estimates may then be used to obtain GLS estimates for the coefficients of (A8a) and (A8b) using weighted least squares. In deriving these GLS estimates for (A8a) and (A8b) the interequation correlation given by (A9f) is still ignored, and asymptotic efficiency cannot be claimed for this GLS procedure. The usual formulas for the standard errors of the coefficients are not appropriate for either the OLS or the GLS procedures described above, except under the null hypothesis of no selection bias.

The correct asymptotic variance-covariance matrix for the OLS case may be obtained as follows. Run OLS regressions (A8a)–(A8b) where $\hat{\lambda}$ is substituted in for λ . Denote by \hat{C} the coefficients of $\hat{\lambda}$ from equation (A8a), where the population value of C is σ_{12}/σ_2 . Let $\hat{\beta}_1$ be the estimated β_1 . Then the corrected standard errors Heckman (1979) derived for the one-limit case can be easily extended to the present two-limit case with the following changes:

- i. Estimate σ_1 by

$$\hat{\sigma}_{11} = \frac{\sum_{i=1}^{I_1} \hat{V}_{1i}^2}{I_1} - \frac{\hat{C}^2}{I_1} \sum_{i=1}^{I_1} (\hat{\xi}_i - \hat{\lambda}_i^2),$$

20 See Heckman (1979).

where $\hat{\xi}_i$ is derived by substituting $\hat{\phi}_{ai}$ and $\hat{\phi}_{bi}$ into (A9g) and \hat{V}_{1i} is the residual term from (A8a).

ii. For calculating $B \psi B'$ for the limiting distribution

$$(I_1)^{\frac{1}{2}} \begin{pmatrix} \hat{\beta}_1 - \beta_1 \\ \hat{C} - C \end{pmatrix} \sim N(0, B \psi B'),$$

Heckman's expression for $B \psi B'$ holds in the two-limit case if the following changes are made:

a. Denote $Z_i = (\phi_{ai}, \phi_{bi})$, and use as Heckman's $\partial \lambda_i / \partial Z_i$ the expression $\partial \lambda_i / \partial Z_i = (\partial \lambda_i / \partial \phi_{ai}, \partial \lambda_i / \partial \phi_{bi})$ where Heckman's equations (15a) and (15b) would now read (in our notation):

$$\frac{\partial \lambda_i}{\partial \phi_{ai}} = \frac{\phi_{ai} f(\phi_{ai})}{F(\phi_{ai}) - F(\phi_{bi})} + \frac{\{f(\phi_{bi}) - f(\phi_{ai})\} f(\phi_{ai})}{\{F(\phi_{ai}) - F(\phi_{bi})\}^2} \quad (15a)$$

and

$$\frac{\partial \lambda_i}{\partial \phi_{bi}} = \frac{-\phi_{bi} f(\phi_{bi})}{F(\phi_{ai}) - F(\phi_{bi})} + \frac{\{f(\phi_{bi}) - f(\phi_{ai})\} f(\phi_{bi})}{\{F(\phi_{ai}) - F(\phi_{bi})\}^2}. \quad (15b)$$

b. Use as Heckman's η_i the expression

$$\eta_i = \{1 + C^2(\xi_i - \lambda_i^2) / \sigma_1^2\}.$$

It should be noted that these corrected standard errors are not valid for the case where (A1a) or (A1b) or both are not reduced from equations. For example, if the offered wage appears on the right-hand side of the hours equations, general expressions for the corrected standard errors are not available for the hours equation.²¹

It should be pointed out that ϕ_{ai} and ϕ_{bi} in (A7) may be defined alternatively as

$$\phi_{ai} = (1/\sigma_2)(a - X_{2i}\beta_2^a), \quad \phi_{bi} = (1/\sigma_2)(b - X_{2i}\beta_2^b), \quad (A11)$$

where some or all of the respective elements of β_2^a and β_2^b may differ. Estimates of the relevant coefficients needed to calculate $\hat{\phi}_{ai}$ and $\hat{\phi}_{bi}$ may still be obtained using probit analysis for the probability that $Y_{2i} > a$ and for the probability that $Y_{2i} > b$. For instance, β_2 in (A1b) may be replaced by $\beta_2^{(Y_2)}$, in which case the functions defining ϕ_{ai} and ϕ_{bi} in (A11) will include $\beta_2^{(Y_2)}$ evaluated at $Y_2 = a$ and at $Y_2 = b$, respectively. If we suspect that β_1 also varies in value over the range of variation for Y_{2i} , we can also replace β_1 in (A1a) by $\beta_1^{(Y_2)}$. In the present application we are interested in determining whether some elements of β_1 and β_2 are functions of a wife's annual hours of work.

Furthermore the range of variation for Y_2 over which Y_1 is observed may be broken up into a sequence of intervals with endpoints $b_1 < b_2 < \dots$; probit analysis

21 Corrected standard errors for a certain type of instrumental variables estimator are considered for simultaneous equations models with selectivity in Lee et al. (1980, equation (24)). Their computational formulas do not apply to our estimation method.

may be repeatedly applied for the probability that $Y_{2i} > b_1$, $Y_{2i} > b_2$, and so forth; and $\beta_2^{b_1}$, $\beta_2^{b_2}$, ... and $\phi_{b_{1i}}$, $\phi_{b_{2i}}$, ... may be estimated. Using these estimates for $\phi_{b_{1i}}$, $\phi_{b_{2i}}$, ... , values for λ_i and \hat{M}_i for each individual in each of the subsamples defined by the endpoints b_1 , b_2 , ... may be obtained from (A6), (A9g) and the formula for M_i . The appropriate OLS or GLS subsample regressions should then yield a piecewise approximation to the underlying response surface.

The method presented for estimating the parameters of a pair of reduced form equations such as (A1a) and (A1b) using data censored at two limits may easily be extended to allow estimation in these same circumstances of the associated pair of structural equations, provided that these structural equations are identified. Suppose, for instance, that the structural system corresponding to the reduced form equations (A1a) and (A1b) is

$$Y_{1i} = X_i\beta_1 + U_{1i} \quad (\text{A12a})$$

and

$$Y_{2i} = \gamma_1 Y_{1i} + X^*\gamma_2 + U_i^*, \quad (\text{A12b})$$

where X_i^* is a vector of exogenous variables, $\gamma = (\gamma_1, \gamma_2)$ is a vector of parameters, and U_i^* is a random disturbance term. Thus (A12a) is identical to (A1a), and (A12b) may be rewritten in the reduced form as (A1b) where X_2 consists of all distinct exogenous variables chosen from X_1 and X^* and $U_{2i} = \gamma_1 U_{1i} + U_i^*$. Assuming the distribution of U_{1i} and U_{2i} thus defined to be bivariate normal, previous results may be used. This extension is relevant in the present application, since we are interested in directly estimating the response of the i th wife's annual hours of work to a given change in her offered wage rate.

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