Concept's International Series in Geography

No. 1. Perspectives in Agricultural Geography (In Five Volumes) : Edited by NOOR MOHAMMAD
No. 2. Perspectives in Geomorphology (In Four Volumes) Edited by : H.S. SHARMA
No. 3. Perspectives in Urban Geography Edited by : C.S. YADAV

1. New Directions in Urban Geography
2. Comparative Urban Research
3. Comparative Urbanization : City Growth and Change
4.A Models in Urban Geography (Theoretical)
4.B Models in Urban Geography (Mathematical)
5. Urban Research Methods : Central Place, Hierarchical and City Size Models
6. Contemporary City Ecology
7. Slums, Urban Decline and Revitalization
8. Contemporary Urban Issues
9. Rural-Urban Fringe
10. Morphology of Towns
11. Cities and Housing
12. Perceptual and Cognitive Image of the City
13. Urban Economics
14. City Planning : Problems and Prospects
15. City Planning : Administration and Participation
16.A Urban Planning and Policies : Reorientation of Policy Norms
16.B Urban Planning and Policies : Cross-cultural Urban Policies

Concept's International Series in Geography No. 3

PERSPECTIVES IN URBAN GEOGRAPHY

VOLUME FOUR-B

MODELS IN URBAN GEOGRAPHY

(Mathematical)

CONTRIBUTOR COPY

Edited by

C. S. YADAV

CONCEPT PUBLISHING COMPANY, NEW DELHI

1987

C. S. Yadav (Ed.) Perspectives in Urban Geography, Vol. 4B, Concept Publishing Company, New Delhi, India, 1987, pp. 595-610.

TWENTY FIVE

ALICE NAKAMURA AND MASAO NAKAMURA

ON ESTIMATING THE STAGES OF THE FAMILY LIFE CYCLE

INTRODUCTION

THE concept of the family life cycle plays an important role in planning and particularly in market segmentation.[2, 3, 4, 5] This article considers analytical and simulation methods to describe the family life cycle. For planning problems where age and sex are sufficient to define the stages of the family life cycle, analytical models are found adequate; for problems where additional demographic variables are required in addition to age and sex to define the stages of the family life cycle, microanalytic simulation[1] is found particularly suitable. The latter problems include demand projections for houses and dairy products as well as for social services such as public schools and day care.

ANALYTICAL METHODS

Suppose the family life cycle consists of the following stages: $I(1)$, Individuals (both male and female), 0–17 years of age; $I(2)$, Males 18–34 years of age; $I(3)$, Females 18–34 years of age; $I(4)$, Males 35–64 years of age; $I(5)$, Females 35–64 years of age and $I(6)$ Individuals (both male and female), over 65 years of age. In order to predict these stages let us define the following variables: $X_j(t)$=number of males in age group j in period t, $Y_j(t)$=number of females in age group

j in period t, where age groups $j=1, 2, \ldots, J$ correspond to the unit period of time (say, a year), $t=0, 1, 2, \ldots, T$ and where J is the total number of age groups and T is the planning horizon. Assuming that there is no migration into and out of the community of our interest, $X_j(t)$ and $Y_j(t)$ satisfy, for $t=0, 1, 2, \ldots, T-1$,

$$(1)\quad X_1(t+1)=\sum_{j=1}^{J} \bar{b}_j(t)\, Y_j(t)$$

$$(2)\quad X_j(t+1)=(1-\bar{d}_{j-1}(t))\, X_{j-1}(t) \qquad j=2, 3, \ldots, J,$$

$$(3)\quad Y_1(t+1)=\sum_{j=1}^{J} b_j(t)\, Y_j(t)$$

$$(4)\quad Y_j(t+1)=(1-d_{j-1}(t))\, Y_{j-1}(t) \qquad j=2, 3, \ldots, J,$$

where $\bar{b}_j(t)$ and $b_j(t)$ are probabilities that a male baby and a female baby are born, respectively, to a female in age group j in period t and that the baby survives to the beginning of period $t+1$, and similarly $\bar{d}_j(t)$ and $d_j(t)$ are probabilities that a male and a female in age group j die, respectively, in period t. Historically male babies out-number female babies by 53% to 47% in North America and we can assume that

$$(5)\quad \bar{b}_j(t)=v\, b_j(t)$$

where v is an estimated constant and assumed to be 1.13 in our case. Then equations (1)–(4) are summarized in matrix form as follows:

$$(6)\quad Z(t+1)=A(t)\, Z(t)$$

where $(7)\quad Z(t)=(X_1(t), X_2(t), \ldots, X_J(t), Y_1(t), Y_2(t), \ldots, Y_J(t))'$

and (8)

$$A(t)=\left[\begin{array}{cccc|cccc}
vb_1(t) & vb_2(t) & \cdots & v\,b_J(t) & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 1-\bar{d}_1(t) & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots & 0 & 1-\bar{d}_2(t) & & \vdots \\
\vdots & \vdots & & \vdots & & & \ddots & \\
0 & 0 & \cdots & 0 & 0 & \cdots & 1-\bar{d}_{J-1}(t) & 0 \\
\hline
b_1(t) & b_2(t) & \cdots & b_J(t) & 0 & \cdots & \cdots & 0 \\
1-d_1(t) & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\
0 & 1-d_2(t) & & \vdots & \vdots & & & \vdots \\
\vdots & & \ddots & & & & & \\
0 & 0 & \cdots\; 1-d_{J-1}(t) & 0 & 0 & \cdots & \cdots & 0
\end{array}\right]$$

and where the prime denotes the transpose of a vector or a matrix. Given $b_j(t)$, $d_j(t)$, $\bar{d}_j(t)$, v and $Z(0)$, we can estimate the sizes of the stages $I(1)-I(6)$ of the family life cycle by (6) for $t=1, 2, \ldots, T$. Furthermore, the stable distribution of these stages as t goes to infinity can be calculated if birth and death rates do not depend on time. (See Appendix-A for the derivation of the stable distribution.) Such stable distribution of the family life cycle could be useful in certain long-range planning problems. It is also possible to consider optimization problems relating to the family stage distribution and such parameters as birth and death rates.[6]

If we have certain socio-economic variables by which individuals in each stage of the family life cycle are classified into a finite number of states, and if the individual is assumed to change his or her state from one time period to the next according to some probabilistic law, say, a Markov process, then we can incorporate it within our framework in the following manner. Suppose an individual in Stage $I(M)$, $M=1, 2, 3, 4, 5, 6$, changes his socio-economic stage from k in period t to l in period $t+1$ with probability $p_{kl}^{(M)}$, where $k, l=1, 2, \ldots, S_M$, and $t=0, 1, 2, \ldots, T-1$, and suppose that we can estimate transition probabilities $p_{kl}^{(M)}$ from data.[7]

The new stages of the family life cycle of our interest are : $II(1L)$, Individuals (both male and female), 0 – 17 years of age, and in state L, where $L=1, 2, \ldots, S_1$. $II(2L)$, Males 18 – 34 years of age, and in state L, where $L=1, 2, \ldots, S_2$. $II(3L)$, Females 18 – 34 years of age, and in state L, where $L=1, 2, \ldots, S_3$. $II(4L)$, Males 35 – 64 years of age, and in state L, where $L=1, 2, \ldots, S_4$. $II(5L)$, Females 35 – 64 years of age, and in stage L, where $L=1, 2, \ldots, S_5$. $II(6L)$, Individuals (both male and female), over 65 years of age, and in stage L, where $L=1, 2, \ldots, S_6$.

Let $q_L^{(M)}(t)$ be the probability that an individual is in Stage $II(ML)$ in period t, where $t=0, 1, 2, \ldots, T$; $M=1, 2, \ldots, 6$, and $L=1, 2, \ldots, S_M$. Then

$$(9)\quad q_L^{(M)}(t+1)=\sum_{l=1}^{S_M} q_l^{(M)}(t)\, p_{lL}^{(M)} \quad L=1, 2, \ldots, S_M$$

where the initial distributions $q_l^{(M)}(0)\ (\geq 0)$ such that

$$(10)\quad \sum_{l=1}^{S_M} q_l^{(M)}(0)=1 \qquad M=1, 2, \ldots, 6$$

are assumed given.

Let $Z_{I(1)}(t)$, $X_{I(2)}(t)$, $Y_{I(3)}(t)$, $X_{I(4)}(t)$, $Y_{I(5)}(t)$ and $Z_{I(6)}(t)$ be the expected sizes of the family life cycle stages $I(1)$–$I(6)$, respectively, derived by (1)–(5) for $t=1, 2, \ldots, T$. Then the sizes of the family life cycle stages $II(1L)$–$II(6L)$ are, respectively,[9] given by $Z_{I(1)}(t)\, q_L^{(1)}(t)$, $X_{I(2)}(t)\, q_L^{(2)}(t)$, $Y_{I(3)}(t)\, q_L^{(3)}(t)$, $X_{I(4)}(t)\, q_L^{(4)}(t)$, $Y_{I(5)}(t)\, q_L^{(5)}(t)$ and $Z_{I(6)}(t)\, q_L^{(6)}(t)$, where L varies over appropriate states. The stable distributions of the stages $II(1L)$–$II(6L)$ can also be derived under certain assum-

ptions and are given in Appendix-B. There are some optimization problems related to the models of this type.[8]

The key assumptions with respect to the models of this type are (1) that the states defined for each of stages $II(1L)$–$II(6L)$ depend on only age and sex as far as individuals' demographic characteristics are concerned ; (2) that individuals within the same family life cycle stage change their states from one time period to the next according to the same set of Markov transition probabilities which must be estimated from data and (3) no in-and-out migration is assumed. The last assumption could be remedied if additional terms corresponding to in-and-out migration were placed on the right-hand side of (6) (or equivalently (1)–(4)) and if we could assume that the migrating individuals are representative in their respective populations. Given appropriate data it is straight forward to relax the second (Markov) assumption above to include higher-order Markov processes or to introduce heterogeneity among the population in the same family life cycle stage or both within the present analytical framework. It is however, not possible to relax the first assumption above to include such demographic variables as the number and ages of childern, the age of spouse and the length of marriage in the present framework as will be seen in the next section.

FAMILY SIZE DISTRIBUTIONS AND ANALYTICAL METHODS

Since many decisions for purchasing public and private goods and services are made by individual households, it is useful to consider the stages of the family life cycle of the following type[8] : *III* (1), Single individuals, 20 – 29 years of age. *III* (2), Married couples, wife 15—34 years of age, no children. *III* (3), Married couples, wife 15—34 years of age, one or more children under 15 years of age. *III* (4), Married couples, wife 15 – 64 years of age, one or more children under 6 years of age. *III* (5), Married couples, wife 15—64 years of age, no children under 6 years of age, one or more children between 6 and 15 years of age. *III* (6), Married couples, wife 35 – 64 years of age, one child under 15 years of age. *III* (7), Married couples, wife 35 – 64 years of age, no children under 15 years of age. *III* (8), Single or widowed individuals, over 65 years of age.

In order to reconstruct historical sizes and predict future sizes of these stages of the family life cycle from existing data, we have to take into account such demographic events as marriage, birth, death and divorce. The econometric methods including simultaneous equations approach to estimate demographic and other socio-economic variables have not been successful.[9,10] This is primarily due to the fact that birth, marriage and divorce rates are very difficult to describe as stable functions of demographic and socio-economic variables. (In fact, Parlett[11] proves, under general conditions, that the marriage rate cannot be described as a function of sex and age distributions of a population only, which indicates that a stable relationship is not likely to be found by regression between the marriage rate and available demographic variables.) It does, therefore, not appear promising to find useful functional relationships between the sizes of the stages *III* (1)—*III* (8) and available demographic and other variables by regression and then use them for estimation and prediction. (Note also that we usually do not have historical series for sizes of the stages.)

Since probabilities for certain demographic events are available, it might be possible to build a descriptive model such as the one consisting of (1)—(6) to predict *III* (1)—*III* (8).

It is straightforward to see that we need to define at least the following state variables : $Xj, p, c1, c2, \ldots, cp\,(t)$ and $Yj, p, c1, c2, \ldots, cp\,(t)$ are the numbers of single, widowed and divorced men and women in age group j, respectively, with p children whose ages are $c1$, $c2, \ldots, cp$ if $p > 0$ in period t ; $XYj1, j2, q, p, c1, c2, \ldots, cp\,(t)$ is the number of married couples in period t where husband and wife are in age groups $j1$ and $j2$, respectively, having married for q periods of time, and with p children whose ages are $c1$, $c2, \ldots, cp$ if $p > 0$ in period t. For all X, Y, and XY variables we assume that all children are alive if $p > 0$ although it is possible to take into account dead children. Let $\bar{b}_j$ and b_j be probabilities that a male baby and a female baby are born to a married woman with p children ($p \geqslant 0$) in age group j in a unit period of time where $\bar{b}_{jp} = v\, b_{jp}$ as in (5) for some constant v. We ignore births to non-married women for simplicity. Let d_{jm} and $\bar{d}_{jm}$ be the probabilities that a male and female in age group j and marital status m($m=1$

single, divorced or widowed, and $m=2$ married) die in a unit period of time, respectively. Let us denote by $W_{j1,\,j2}$ the joint probability that a single, divorced or widowed man in age group $j1$ and a single, divorced or widowed woman in age group $j2$ get married in a unit period of time. Finally, let dv_q be the probability that a couple which has been married for q periods of time gets divorced in a period of time. Wife is assumed to take over children in case of divorce.

Given that we have estimated probabilities for v, b_{jp}, d_{jm}, $\bar{d}_{jm}$, $w_{j1,\,j2}$, and dv_q, if we could describe the state variables $Xj, p, c1, c2, \ldots, cp\,(t+1)$, $Yj, p, c1, c2, \ldots, cp\,(t+1)$ and $XYj1, j2, q, p, c1, c2, \ldots, cp\,(t+1)$ in terms of their values in period T and v, b_{jp}, d_{jm}, $\bar{d}_{jm}$, $W_{j1,\,j2}$, and dv_q as a recursive system of linear (and possibly nonlinear) equations, we would be able to estimate the life cycle stages *III* (1)—*III* (8). This task, though not impossible, does not appear feasible. This seems to suggest that we need to devise a different method to derive the state variables X, Y, and XY over time given v, b_{jp}, d_{jm}, $\bar{d}_{jm}$, $W_{j1,\,j2}$, and dv_q.

A SIMULATION METHOD TO DERIVE FAMILY SIZE DISTRIBUTIONS AND A NUMERICAL EXAMPLE

In order to derive family size distributions of a population we use microanalytic simulation which is discussed in detail elsewhere.[1] In this method of simulation a population consisting of individuals is simulated over time probabilistically. The size of the population to be simulated is scaled down to, say, one-500th of the real one. Associated with each individual in the simulation population is the following record of characteristics : sex*, date of birth*, marital status (single, married, divorced, widowed), parity**, number of living children**, number of marriages, date of current marital status, date of birth of spouse, date of last live birth**, and number of events, where characteristics with an asterisk are not subject to change once assigned a value and characteristics with two asterisks are only valid for females. Parity represents the number of children ever born to a female and may be larger than the number of living children of the female. Associated with each child (both living and dead) is a child trailer which consists of the following characteristics: date of birth of the child, date of death of the child, if applicable, and sex of the child. The child trailer exists only if parity > 0. While a population is moved forward over time during a simulation run, an individual is subject to risks of the following events: birth, divorce, death, marriage, widowhood and death of child. Associated with each event that actually happens to an individual is an event trailer which consists of type of the event, date of occurrence of the event, marital status of the individual just prior to the event, and additional information about the event as follows: interval since previous birth (for birth), length of marriage in months (for divorce), age at death in month (for death), interval since last change in marital status (for marriage) and interval since marriage (for widowhood). We can use a machine-readable census file which consists of records of individuals randomly chosen from population of our interest for the initial year (usually the census year and $t=0$). If such a file does not exist, it can be created by a program[12] using published aggregete data on age and sex-marital distributions of the population.

We move forward the population from year t to year $t+1$ by subjecting each individual to the risks of possible events which can take place with specified probabilities which are functions of the individual's characteristics at year t. This is then repeated for as many years as desired. At the end of each simulated year we collect information regarding family life cycle stages from the simulated population and print it out.

It is seen that this method of simulation provides the values for the state variables X, Y and XY over time given the same set of data and can be viewed as an approximate method to the analytical method discussed in the previous section.

We applied this simulation model to derive the stages of the family life cycle *III* (1)=*III* (8) for the population of Alberta, a province of Canada, for 1961-1971. The probabilities of birth, death, marriage and divorce required for simulation as well as the initial population used (which is equal to one-500th of the real population created in conformity with published cross-tabulations for Alberta based on the 1961 Canadian census) are identical to the ones reported previously.[1] Over a historical time period our microsimulation model can be used in either a tracking or non-tracking mode. Simulation without tracking over a historical period provides evidence concerning the ability of the model to generaté unconditional forecasts over some future period. Simulation with tracking allows us to incorporate into our simulation all available historical information about the numbers of births, deaths, and marriages which actually occurred in Alberta in each year. This allows us to generate historical time series which, in general, will be closer to the actual time paths of the population of Alberta with usually a substantial reduction in variance of the generated series.

Five independent runs were performed with tracking for the historical period 1961-1971. The simulation results for the 8 stages of the family life cycle *III* (1)$-$*III* (8) are shown in Table 25.1. We are presently planning to explain, together with other explanatory variables, historical demands for housing and day care; derived relationships between such demands and explanatory variables could be used to forecast future demands. We also note that yearly estimates and census figures are only available from Statistics Canada for stages *III* (1) and *III* (8) (but not for *III* (2)$-$*III* (7)) of the family life cycle defined in this manner.

In closing several points should be noted. First, it would be a trivial matter to change the cross-classification part of our program so as to produce forecasts for some other definition of the family life cycle. It is also straightforward to add certain socio-economic characteristics required in certain marketing problems. Secondly, improved probability estimates for forecasting annual births, deaths, marriages, or divorces can easily be incorporated into our simulation system. Information on net migration when available, can also be used to appropriately adjust the simulation population. This may be particularly important in preparing regional forecasts. Finally, in addition to providing forecasts over future time periods, and otherwise unavailable historical time series to be used for research purposes, a forecasting model of the sort we have presented allows vital statistics data available with a relatively short time lag to be used to generate preliminary population estimates prior to the release of census publications and official population forecasts.

TABLE 25.1 : **Estimated numbers of families in Alberta by stage of the family life cycle**

Year	Stage of Family Life Cycle							
	1	2	3	4	5	6	7	8
1961	136.0	37.0	184.0	228.0	126.0	170.0	159.0	78.0
	(0.00)[a]	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
1962	134.4	31.8	188.2	239.0	122.6	173.4	164.8	79.2
	(3.05)	(1.79)	(1.79)	(1.41)	(2.70)	(1.52)	(1.79)	(2.78)
1963	131.8	29.2	190.4	248.4	126.2	184.2	163.8	83.2
	(3.49)	(3.96)	(4.16)	(5.03)	(3.83)	(3.03)	(1.92)	(4.97)
1964	135.4	27.6	191.4	261.6	118.8	189.0	167.0	83.4
	(4.93)	(3.91)	(2.88)	(4.72)	(4.55)	(3.08)	(2.55)	(6.66)
1965	128.4	26.6	196.2	266.0	121.8	191.6	172.0	86.4
	(4.50)	(6.54)	(6.53)	(9.56)	(5.26)	(5.18)	(5.10)	(6.80)
1966	128.6	28.6	195.2	265.0	127.4	197.2	177.2	88.8
	(2.61)	(6.07)	(7.36)	(9.98)	(4.39)	(4.15)	(5.76)	(8.32)
1967	131.2	33.8	193.8	264.6	131.4	202.2	177.6	91.0
	(4.15)	(7.69)	(2.49)	(3.91)	(2.19)	(3.27)	(8.05)	(7.31)
1968	131.6	31.8	195.4	267.8	134.2	204.8	185.8	95.6
	(6.35)	(4.60)	(7.16)	(6.30)	(3.03)	(3.56)	(8.44)	(6.11)
1969	129.8	35.6	202.0	265.2	142.4	205.6	192.0	98.4
	(1.79)	(5.46)	(3.39)	(5.63)	(4.34)	(5.94)	(10.34)	(5.50)
1970	128.0	42.4	202.8	262.6	145.0	204.8	201.0	100.4
	(5.61)	(7.13)	(6.06)	(7.16)	(3.24)	(4.97)	(11.04)	(3.21)
1971	130.4	43.6	211.8	268.2	154.8	211.2	202.0	104.2
	(4.72)	(6.23)	(6.83)	(8.05)	(4.60)	(5.45)	(9.22)	(6.14)

[a] Numbers in parentheses are standard deviations. The standard deviations for our figures for 1961 are all zero since the same initial population was used for all simulation runs.

(In Canada, for instance, these official estimates and the census data are usually made available several years after the data on which they are based was collected.)

CONCLUSIONS

We have shown that while they are useful for estimating the family life cycle stages defined only in terms of age and sex, analytical models become too complex to handle once family sizes enter to define the family life cycle stages. In this paper microanalytic simulation is used to estimate such family life cycle stages and numerical results are also given. Simulation approach to estimate the sizes of family life cycle stages appears promising since it can be easily adapted to a variety of definitions of the family life cycle stages.

APPENDIX A

DERIVATION FOR A STABLE AGE-SEX DISTRIBUTION

Let us denote by $Z_j = \lim_{t\to\infty} (x_j(t) / \sum_{i=1}^{J} x_i(t))$ and $Z_{J+j} = \lim_{t\to\infty} (y_j(t) / \sum_{i=1}^{J} y_i(t))$ the stable age distributions of men and women in age group j ($j=1, 2, \ldots, J$), respectively. We let $Z_J^* = Z_{J+j}$ in the following. Assuming that birth and death rates do not depend on time, the Z_j^* satisfy[6]

$$
\text{(AI)}\quad
\begin{aligned}
\lambda Z_1^* &= b_1 Z_1^* + b_2 Z_2^* + \ldots + b_{J-1} Z_{J-1}^* + b_J Z_J^* \\
\lambda Z_2^* &= (1-d_1) Z_1^* \\
\lambda Z_3^* &= (1-d_2) Z_2^* \\
&\vdots \\
\lambda Z_J^* &= (1-d_{J-1}) Z_{J-1}^*
\end{aligned}
$$

where $\sum_{j=1}^{J} Z_j^* = 1$, $Z_j^* \geqslant 0$ ($j=1, 2, \ldots, J$), and $\lambda \geqslant 0$. The λ has the following interpretation : $\lambda = \lim_{t\to\infty} (\sum_{j=1}^{J} y_j(t+1) / \sum_{j=1}^{J} y_j(t))$, that is, λ represents the rate of population growth of females. Once (A1) is solved ior Z^* and λ, then the Z_j are seen to be given by solving

$$\lambda Z_1 = v\,(b_1 Z_1 + b_2 Z_2 + \ldots + b_J Z_J)$$

$$\text{(A2)} \qquad \lambda Z_2 = (1-\bar{d}_1)\, Z_1$$

$$\vdots$$

$$\lambda Z_J = (1-\bar{d}_{J-1})\, Z_{J-1}.$$

It can be shown that both (A1) and (A2) have unique solutions and hence these can be solved by any numerical method to solve non-linear equations[13] given values for b_j, v, d_j and $\bar{d}_j$. The fact that both male and female populations have the same population growth rate is proved as follows.

$$\text{Let} \quad \lambda_1 = \lim_{t\to\infty} \Big(\sum_{j=1}^{J} x_j\,(t+1) \Big/ \sum_{j=1}^{J} x_j\,(t)\Big).$$

Then

$$\lambda_1/\lambda = \lim_{t\to\infty} \Big(\sum_{j=1}^{J} x_j\,(t+1) \Big/ \sum_{j=1}^{J} x_j\,(t)\Big) \lim_{t\to\infty} \Big(\sum_{j=1}^{J} y_j\,(t) \Big/ \sum_{j=1}^{J} y_j\,(t+1)\Big)$$

$$= \lim_{t\to\infty} \Big(\sum_{j=1}^{J} x_j\,(t+1) \Big/ \sum_{j=1}^{J} y_j\,(t+1)\Big) \lim_{t\to\infty} \Big(\sum_{j=1}^{J} y_j\,(t) \Big/ \sum_{j=1}^{J} x_j\,(t)\Big) = 1$$

proving $\lambda_1 = \lambda$.

Rearranging equations in (A1) and (A2) we get

$$\text{(A3)} \qquad \lambda - 1 = (b_1 - d_1)\, Z_1^* + (b_2 - d_2)\, Z_2^* + \ldots + (b_j - d_{J-1})\, Z_{J-1}^* + (b_j - 1)\, Z_J^*$$

and

$$\text{(A4)} \qquad \lambda - 1 = (vb_1 - \bar{d}_1)\, Z_1 + (vb_2 - \bar{d}_2)\, Z_2 + \ldots + (vb_j - \bar{d}_{J-1})\, Z_{J-1} + (b_J - 1)\, Z_J$$

which explain how each age group in each sex contributes to the population growth rate.

APPENDIX B

DERIVATION FOR A STABLE AGE-SEX-MARKOV-STATE DISTRIBUTION

Suppose the finite Markov chain with transition probabilities $p_{k1}^{(M)}$ k, $l=1, 2, \ldots, S_M$ is irreducible and aperiodic for $M=1, 2, \ldots, 6$. Then stationary probabilities $\pi_1^{(M)}$, $l=1, 2, \ldots, S_M$, are given by[14]

$$\pi_1^{(M)} = \sum_{k=1}^{S_M} \pi_k^{(M)}\, p_{k1}^{(M)} \quad 1=1, 2, \ldots, S_M$$

$$\sum_{l=1}^{S_M} \pi_1^{(M)} = 1.$$

Let us denote by $Z_{I(1)}$, $x_{I(2)}$, $y_{I(3)}$, $x_{I(4)}$, $y_{I(5)}$ and $Z_{I(6)}$ the appropriate stable distributions derived from Appendix-A corresponding to $Z_{I(1)}\,(t)$, $x_{I(2)}\,(t)$, $y_{I(3)}\,(t)$, $x_{I(4)}\,(t)$, $y_{I(5)}\,(t)$ and $Z_{I(6)}\,(t)$. Then the stable distribution of the stages *II*(1L)—*II*(6L) are given by

$$Z_{I(1)}\,\pi_L^{(1)},\; x_{I(2)}\,\pi_L^{(2)},\; y_{I(3)}\,\pi_L^{(3)},\; x_{I(4)}\,\pi_L^{(4)},\; y_{I(5)}\,\pi_L^{(5)} \text{ and } Z_{I(6)}\,\pi_L^{(6)}.$$

REFERENCES

1. A. Nakamura and M. Nakamura, On microanalytic simulation and its applications to population projection. *Journal of the Operational Research Society*, 29, No. 4, 1978, 349-360.
2. J.R. Bomball et al., Forecasting the stage 2 of the family life cycle. *Journal of Business*, 1, 1975, 65-73.
3. L.E. Boone and D.L. Kurtz, *Contemporary Marketing*. Dryden Press, Hinsdale, 111, 1974.
4. R.E. Frank et al., *Market Segmentation*. Prentice-Hall, Englewood Cliffs, N.J., 1972.

5. P.K. Kotler, *Marketing Management: Analysis, Planning and Control*, Prentice-Hall, Englewood Cliffs, N.J., 1972.
6. M. Nakamura, Some programming problems in population projection. *Operations Research* 21, 1973, 1048-1062.
7. T.C. Lee et al., Estimating the Parameters of the Markov Probability Model from Aggregate Time Series Data. North-Holland, Amsterdam, 1970.
8. M. Nakamura, On a class of optimization problems with a specified growth pattern, *Management Science*, 20, 1973, 236-239.
9. M. Nerlove and T.P. Schultz, Love and Life between the Consensus : A Model of Family Decision Marking in Puerto Rico, 1950-1960. Rand : Corporation, Santa Monica, Calif., 1970.
10. N.B Ryder, Comment on 'A new approach to the economic theory of fertility behavior.' *Journal of Political Economy*, 81, Part II, 1973, 65-69.
11. B. Parlett, Can there be a marriage function ? in T.N.E. Greville (Ed.), *Population Dynamics*. Academic Press, New York, 1972,
12. U.S. Department of Health, Education and Welfare User's Manuel for Popsim. National Center for Health Statistics, Rockville, Md., 1973.
13. T.L. Saaty and J. Bram, *Nonlinear Mathematics*, McGraw-Hill, New York, 1964.
14. E. Cinlar, *Introduction to Stochastic Processes*. Prentice-Hall, Englewood, New Jersey, 1975.

TWENTY SIX

WILMER M. HARPER AND JOEL A. DIEMER

QUALITY OF LIFE ASSESSMENT
A Methodology for Intraregional Comparisons

INTRODUCTION

THE realization and maintenance of some minimum standard of living or quality of life (Q of L) is integral to the social welfare goals of virtually all contemporary societies. This is amply manifest in the actions of national, sub-national and local governments as they propose and initiate programs to reduce unemployment, improve public services, etc. However, with few exceptions the measurement of the efficiency of such programs in achieving improvements in perceived well-being and Q of L has been particularly difficult. Understandably this is often a particularly distressing situation for target groups and program administrators, not to mention responsible politicians. This is, of course, particularly true during periods of economic downturn when demands are highest and resources most scarce.

Most developed societies are entering what Angus Campbell has called the era of "psychological man". When researchers at the Institute for Social Research asked individuals "What does quality of life mean to you—that is, what would you say the overall quality of life depends on ?", the following areas in order of frequency were reported : (1) economic security, (2) family life, (3) personal strengths, (4) friendships, (5) attractiveness of the physical environment [11, p. 4]. As society becomes more affluent and the general standard of living rises,