

Assignment 5 Question 3  
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First, the integration by substitution and parts

$$\int_{-1}^0 (x-1)\sin(3x-3)dx$$

use the integration by substitution

Assume  $(x-1)$  is  $a$ , therefore  $dx=da$

Therefore, the function becomes  $\int_{-1}^0 a\sin(3a)dx$

use integration by parts

Then we assume  $U = a$ ,  $du = 1$ ,  $dv = \sin 3a$ ,  $v = \frac{-\cos 3a}{3}$

$$\begin{aligned} \text{So } \int_{-1}^0 (x-1)\sin(3x-3)dx &= a * \frac{-\cos 3a}{3} \Big|_{-1}^0 - \int_{-1}^0 \frac{-\cos 3a}{3} * da \\ &= \frac{\cos(-3)}{3} + \frac{1}{9} + \frac{\sin(-3)}{9} = -0.23456 \end{aligned}$$

Second, the integration with substitution and partial fraction

$$\int_0^1 \frac{2u(u^4 - 39u^2 + 5)}{(u^2 - 4)^2(u^4 + 3)} * du$$

use integration by substitution

We assume  $u^2 = x$ , so  $2u*du=dx$

$$\text{therefore } = \int_0^1 \frac{2u(x^2 - 29x + 5)}{(x-4)^2(x^2+3)} * \frac{dx}{2u}$$

$$= \int_0^1 \frac{x^2 - 29x + 5}{(x-4)^2(x^2+3)} dx$$

use integration by partial

$$= \int_0^1 \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x^2+3} dx$$

$$= \int_0^1 \frac{1}{x-4} + \frac{5}{(x-4)^2} + \frac{-x+2}{x^2+3} dx$$

$$= \int_0^1 \frac{1}{x-4} - \frac{5}{(x-4)^2} - \frac{x}{x^2+3} + \frac{2}{x^2+3} dx$$

$$= -\frac{5}{12} + \frac{\pi}{3\sqrt{3}} \ln\left(\frac{3\sqrt{3}}{8}\right)$$

Third, the integration with partial and parts

$$\int_0^1 \log(x-1) \frac{-2}{(x+2)^2}$$

using integration by parts

We assume  $u = \log(x-1)$ ,  $du = \frac{1}{x-1}$ ,  $v = \frac{x+4}{x+2}$ ,  $dv = \frac{-2}{(x+2)^2}$

$$\begin{aligned}
& \int_0^1 \log(x-1) \frac{-2}{(x+2)^2} = \log(x-1) * \frac{x+4}{x-2} \Big|_0^1 - \int_0^1 \frac{x+4}{x+2} \frac{1}{x-1} \\
& \text{using integration by partial} \\
& \log(x-1) * \frac{x+4}{x+2} \Big|_0^1 - \int_0^1 \frac{x+4}{(x+2)(x-1)} \\
& = \log(x-1) * \frac{x+4}{x+2} \Big|_0^1 - \int_0^1 \frac{A}{x+2} + \frac{B}{x-1} \\
& A=2, B=3 \\
& = \log(x-1) * \frac{x+4}{x+2} \Big|_0^1 - \int_0^1 \frac{2}{x+2} + \frac{3}{x-1} \\
& = \frac{1}{3(\ln(\frac{9}{4}) - i\pi)}
\end{aligned}$$