

Integral Calculus

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Examples on dot product

Example 0.1. Compute the dot product of the following vectors.

- $\mathbf{u} = 2\mathbf{i} - 6\mathbf{j} = \langle 2, -6, 0 \rangle$ and $\mathbf{v} = 12\mathbf{k} = \langle 0, 0, 12 \rangle$

solution:

$$\mathbf{u} \cdot \mathbf{v} = 2 \cdot 0 + (-6) \cdot 0 + 0 \cdot 12 = 0$$

- $\mathbf{u} = \sqrt{3}\mathbf{i} + \mathbf{j} = \langle \sqrt{3}, 1 \rangle$ and $\mathbf{v} = \mathbf{j} = \langle 0, 1 \rangle$

solution:

$$\mathbf{u} \cdot \mathbf{v} = \sqrt{3} \cdot 0 + 1 \cdot 1 = 1$$

Example 0.2. Find the angle between \mathbf{u} and \mathbf{v} (use $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| \cdot |\mathbf{v}| \cos \theta$), then conclude. (They are parallel if and only $\mathbf{u} \cdot \mathbf{v} = \pm |\mathbf{u}| \cdot |\mathbf{v}|$, which means $\theta = 0$ or $\theta = \pi$. They are orthogonal if and only $\mathbf{u} \cdot \mathbf{v} = 0$ which means $\theta = \pi/2$).

- $\mathbf{u} = \sqrt{3}\mathbf{i} + \mathbf{j} = \langle \sqrt{3}, 1 \rangle$ and $\mathbf{v} = \mathbf{j} = \langle 0, 1 \rangle$

solution:

$$|\mathbf{u}| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2; \quad |\mathbf{v}| = \sqrt{(0)^2 + 1^2} = 1$$

$$\mathbf{u} \cdot \mathbf{v} = \sqrt{3} \cdot 0 + 1 \cdot 1 = 1$$

$$\text{So, we have: } 1 = 2 \cos \theta \implies \cos \theta = \frac{1}{2} \implies \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

\mathbf{u} and \mathbf{v} are not parallel nor orthogonal.

- $\mathbf{u} = \mathbf{i} + \mathbf{j} = \langle 1, 1 \rangle$ and $\mathbf{v} = \mathbf{i} - \mathbf{j} = \langle 1, -1 \rangle$

solution:

$$|\mathbf{u}| = \sqrt{1^2 + 1^2} = \sqrt{2}; \quad |\mathbf{v}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\mathbf{u} \cdot \mathbf{v} = 1 \cdot 1 + 1 \cdot (-1) = 0$$

$$\text{So, we have: } 0 = \sqrt{2} \cos \theta \implies \cos \theta = 0 \implies \theta = \cos^{-1}(0) = \frac{\pi}{2}$$

\mathbf{u} and \mathbf{v} are orthogonal.

- $\mathbf{u} = 2\mathbf{i} + \mathbf{j} = \langle 2, 1 \rangle$ and $\mathbf{v} = -4\mathbf{i} - 2\mathbf{j} = \langle -4, -2 \rangle$

solution:

$$|\mathbf{u}| = \sqrt{2^2 + 1^2} = \sqrt{5}; \quad |\mathbf{v}| = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$\mathbf{u} \cdot \mathbf{v} = 2 \cdot (-4) + 1 \cdot (-2) = -10$$

$$\text{So, we have: } -10 = \sqrt{5} \cdot 2\sqrt{5} \cos \theta \implies -10 = 10 \cos \theta \implies \cos \theta = -1 \implies \theta = \cos^{-1}(-1) = \pi$$

\mathbf{u} and \mathbf{v} are parallel.