Homework week 2 solutions

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0.1 Parallel or orthogonal?

Which of these distinct planes are parallel and which are orthogonal? (**5 points**)

$$\mathcal{Q} : x - \frac{3}{2}y + 3z = 6 \qquad \mathcal{R} : -2x + 3y - 6z = 1 \qquad \mathcal{S} : 3x + 4y + z = 3$$

Solution:

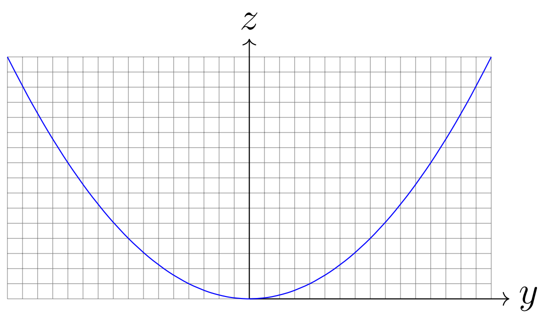
- Normal vectors: $\mathbf{n}_{\mathcal{Q}} = \langle 2, -\frac{3}{2}, 3\rangle$, $\mathbf{n}_{\mathcal{R}} = \langle -2, 3, -6\rangle$ and $\mathbf{n}_{\mathcal{S}} = \langle 3, 4, 1\rangle$ (**1 point**)
- $\mathbf{n}_{\mathcal{R}} = -2\mathbf{n}_{\mathcal{Q}} \Longrightarrow \dfrac{-2}{1} = \dfrac{3}{-\frac{3}{2}} = \dfrac{-6}{3} = -2 \Longrightarrow \mathbf{n}_{\mathcal{R}}$ is parallel to $\mathbf{n}_{\mathcal{Q}}$ (**1 point**)
- $\mathbf{n}_{\mathcal{Q}} \cdot \mathbf{n}_{\mathcal{S}} = 0 \Longrightarrow 1 \cdot 3 + (-\frac{3}{2}) \cdot 4 + 3 \cdot 1 = 0 \Longrightarrow \mathbf{n}_{\mathcal{Q}}$ is orthogonal to $\mathbf{n}_{\mathcal{S}}$ (**1 point**)
- $\mathbf{n}_{\mathcal{R}} \cdot \mathbf{n}_{\mathcal{S}} = 0 \Longrightarrow -2 \cdot 3 + 3 \cdot 4 + (-6) \cdot 1 = 0 \Longrightarrow \mathbf{n}_{\mathcal{R}}$ is orthogonal to $\mathbf{n}_{\mathcal{S}}$ (**1 point**)
- Since $\mathcal{Q}$ and $\mathcal{R}$ are parallel they are both orthogonal to $\mathcal{S}$ (**1 point**)

0.2 Traces and Level Curves

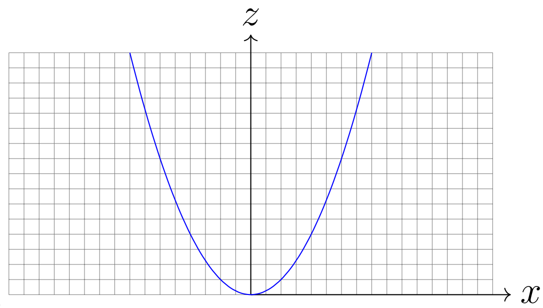
Given the function $4z = x^2 + \dfrac{y^2}{4}$.

Exercise 0.1. Explain briefly what the shapes of $xy-$trace, $yz-$trace and $xz-$trace of the function $z = f(x, y)$ are, such as line, circles and parabolas. Sketch them. (**3 points**)
Solution:
First, $z = \dfrac{x^2}{4} + \dfrac{y^2}{16}$

- $xy-$trace: $z = f(x, y)$ has its minimum value at $z = 0$. So the $xy-$trace is just a point (0,0,0). (**1 point**)



- $yz-$trace: $z = \dfrac{y^2}{16}$. It's an equation of a parabola. (**1 point**)



- $xz-$trace: $z = \frac{x^2}{4}$. It's an equation of a parabola. (**1 point**)

Exercise 0.2. Explain briefly what the shape of the level curves of the function $z = f(x, y)$ is, such as line, circles and parabolas. Sketch the level curves $f(x, y) = z_0$ with $z_0 = 1$ and $z_0 = 2$ (**2 points**)
Solution:

- The level curves are ellipses for which $a < b$. (**1 point**)
- Figures: the left figure gives the solution for $z_0 = 1$ and $z_0 = 2$. (**1 point**)