Math 104 section 108 Homework 1 (Solutions)

Daniel Rakotonirina

September 19, 2017

1 Exponential/Logarithmic functions

Exercise 1.1. Solve $(3^x)^2 = \frac{1}{5}$ for x. (1 mark)

Solution 1.1.

$$2x = \log_3\left(\frac{1}{5}\right) = -\log_3(5) \qquad \text{(0.5 mark)}$$

$$x = -\frac{1}{2}\log_3(5) \qquad \text{(0.5 mark)}$$

Exercise 1.2. Solve $9^{x^2-3x+\frac{3}{2}} = \frac{1}{3}$ for x. (1 mark)

Solution 1.2.

$$\log_3\left(3^{2(x^2-3x+\frac{3}{2})}\right) = \log_3\frac{1}{3}$$

$$x^2 - 3x + \frac{3}{2} = \frac{\log_3\frac{1}{3}}{\log_3(3^2)} = \frac{\log_3 1 - \log_3 3}{2} = \frac{0-1}{2} \qquad \text{(0.5 mark)}$$

$$x^2 - 3x + \frac{3}{2} + \frac{1}{2} = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x = 1 \qquad x = 2 \qquad \text{(0.5 mark)}$$

Exercise 1.3. Solve $2^{\log_3(x+2)} = 5$ for x. (1 mark)

Solution 1.3.

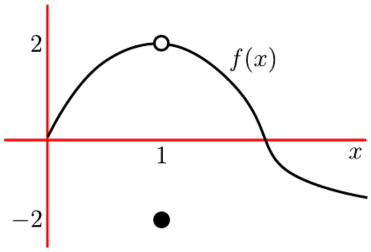
$$\log_2 2^{\log_3(x+2)} = \log_2 5$$

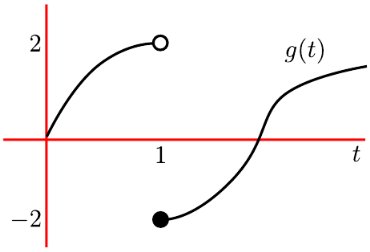
$$\log_3(x+2) = \log_2 5$$

$$3^{\log_3(x+2)} = 3^{\log_2 5} \qquad \text{(0.5 mark)}$$

$$x + 2 = 3^{\log_2 5}$$

$$x = 3^{\log_2 5} - 2 \qquad \text{(0.5 mark)}$$





2 Limits

Exercise 2.1. Given the following graphs.

Compute the following limits:

1. $\lim_{x \to 1^-} f(x) = 2$ (0.5 mark)

2. $\lim_{x \to 1^+} f(x) = 2$ (0.5 mark)

3. $\lim_{t \to 1^-} g(t) = 2$ (0.5 mark)

4. $\lim_{t \to 1^+} g(t) = -2$ (0.5 mark)

5. $\lim_{t \to 1} g(t) = DNE$ (0.5 mark)

3 Business problem – you must show your work

Exercise 3.1. A manufacturer sells 50 tables a month at the price of \$300 each. For each \$8 decrease in price, he can sell 2 more tables. Their factory costs \$5,000 per month to operate and each table costs an additional \$50 to make. Note: in this problem you are ONLY setting up the equations. You do NOT have to solve for break even values or any optimal production values.

1. Find the linear demand equation for the tables. Use the notation p for the unit price and q for the monthly demand. (1.5 mark)

   Solution 3.1. A data point is $(q, p) =$ (quantity, price). So two points are (50, 300) and (52, 292)

$$p = mq + b \tag{1}$$

$$300 = 50m + b \tag{2}$$

$$292 = 52m + b \qquad \text{(0.5 mark)} \tag{3}$$

   We can solve equation (1) for b to find $b = 300 - 50m$. We can then plug this in equation (3) to find m:

$$292 - 52m = 300 - 50m \tag{4}$$

$$292 - 300 = -50m + 52m \tag{5}$$

$$-8 = 2m \tag{6}$$

$$m = -4 \Longrightarrow b = 500 \qquad \text{(0.5 mark)} \tag{7}$$

   The answer is: $p = -4q + 500$ (0.5 mark)

2. Find the cost function, $C = C(q)$, for producing q tables per month. (1 mark)

Solution 3.2. No work needed

$$C(q) = 5000 + 50q \qquad (1 \text{ mark})$$

3. Find the monthly revenue function, $R = R(q)$. (1 mark)

Solution 3.3.

$$R(q) = p \cdot q = (-4q + 500) \cdot q \qquad (0.5 \text{ mark})$$

$$R(q) = -4q^2 + 500q \qquad (0.5 \text{ mark})$$

4 Limit process

Exercise 4.1. Use the definition of the derivative, i.e. *the limit process*, to find the slope of the tangent line to the graph of $y = \sqrt[3]{x}$ at $x = 8$. Please put a box around your final answer. (1 mark + **bonus: 2 marks**)

Solution 4.1.

$$\begin{aligned}
f'(2) &= \lim_{x\to 2} \frac{f(x) - f(8)}{x-8} = \lim_{x\to 2} \frac{\sqrt[3]{x} - \sqrt[3]{8}}{x-8} \text{ (1 mark)} \\
&= \lim_{x\to 2} \frac{\sqrt[3]{x} - 2}{x-8} \\
&= \lim_{x\to 2} \left(\frac{\sqrt[3]{x} - 2}{x-8} \, \frac{x^{2/3} + 2^2 + 2x^{1/3}}{x^{2/3} + 2^2 + 2x^{1/3}} \right) \\
&= \lim_{x\to 2} \left(\frac{(x^{1/3} - 2)(x^{2/3} + 2^2 + (2x)^{1/3})}{(x-8)(x^{2/3} + 2^2 + 2x^{1/3})} \right) \\
&= \lim_{x\to 2} \left(\frac{x-8}{(x-8)(x^{2/3} + 2^2 + 2x^{1/3})} \right) \\
&= \lim_{x\to 2} \left(\frac{1}{x^{2/3} + 2^2 + 2x^{1/3}} \right) \text{ (1 mark)} \\
&= \frac{1}{8^{2/3} + 2^2 + 2 \times 8^{1/3}} \\
&= \frac{1}{(2^3)^{2/3} + 2^2 + 2 \times (2^3)^{1/3}} \\
&= \frac{1}{2^2 + 2^2 + 2 \times 2} \\
&= \frac{1}{12} \text{ (1 mark)}
\end{aligned}$$